Technological Leadership and Persistence of Monopoly under Endogenous Entry: Static *versus* Dynamic Analysis

Supplementary Appendix: First-best

Eugen Kováč [a,b,c] Viatcheslav Vinogradov [b,d]

Krešimir Žigić [b,e]

March 2010

In this appendix we consider the first-best situation, in which a social planner or a government can, besides R&D investments, also determine firms' entry decisions and outputs, that is, the social planner takes all strategic decisions. Clearly, in this case, the planner would ban the entry of any followers in order to avoid the duplication of fixed costs. Thus, we are left with a single firm being active — the leader. In what follows, we consider two cases: one in which the leader's overall profit can also be negative and another one in which we impose an additional requirement that the leader's overall profit is non-negative. This requirement can be interpreted as a participation constraint.

We first analyze the case without the participation constraint, i.e., when the leader's profit can attain negative values (so called "unconstrained first-best"). The social planner then solves the following optimization problem:

$$I_{FB} = \max_{p,z} \int_0^{+\infty} \big[\big(A - p(t)\big)\big(p(t) - c_0(t)\big) - z^2(t) - F + \tfrac{1}{2}\big(A - p(t)\big)^2\big] e^{-rt}\, dt, \quad (FB)$$

$$\text{subject to}$$
$$\dot{c}_0(t) = \mu[\bar{c} - c_0(t) - \sqrt{g}\, z(t)],$$
$$c_0(0) = \bar{c}.$$

Compared to the optimization problems considered in the paper where a particular market structure determines the relation between the leader's costs and its output (or price), here the price is "free" and it becomes a second control. Furthermore, the problem (FB) is

[a]Department of Economics, University of Bonn, Adenauerallee 24-26, 53113 Bonn, Germany

[b]CERGE-EI, Charles University Prague and Academy of Sciences of the Czech Republic, Politických vězňů 7, 11121 Prague, Czech Republic

[c]Corresponding author. E-mail: eugen.kovac@uni-bonn.de; URL: www.uni-bonn.de/~kovac.

[d]E-mail: viatcheslav.vinogradov@cerge-ei.cz.

[e]E-mail: kresimir.zigic@cerge-ei.cz.

particularly simple, as the price $p(t)$ does not appear in any of the constraints. Thus, $p(t)$ will be chosen to maximize the instantaneous welfare at each point in time. This yields marginal cost pricing: $p(t) = c_0(t)$. Then $q(t) = A - c_0(t)$ and we can rewrite the system in terms of the leader's output

$$I_{FB} = \max_z \int_0^{+\infty} [\tfrac{1}{2}q^2(t) - z^2(t) - F]e^{-rt}\,dt, \qquad (FB')$$

$$\text{subject to}$$

$$\dot{q}(t) = \mu[(A - \bar{c}) - q(t) + \sqrt{g}\,z(t)],$$

$$q(0) = A - \bar{c}.$$

This system can be solved by the same procedure as we use in the paper (for example, in the case of accommodation). In particular, we obtain the steady-state

$$z^*_{BF} = \frac{\sqrt{g}}{2\rho - g}(A - \bar{c}), \qquad q^*_{BF} = \frac{2\rho}{2\rho - g}(A - \bar{c})$$

and the following joint dynamics of z and q:

$$z_{FB}(t) = z^*_{FB} - \frac{g}{2\rho - g}(A - \bar{c}) \cdot \frac{\rho + 1 - \sqrt{(\rho+1)^2 - 2g}}{\sqrt{g}}\, e^{\lambda_{FB} t},$$

$$q_{FB}(t) = q^*_{FB} - \frac{g}{2\rho - g}(A - \bar{c})\, e^{\lambda_{FB} t},$$

where $\lambda_{FB} = \frac{1}{2}\mu[\rho - 1 - \sqrt{(\rho+1)^2 - 2g}]$. We can easily see that the steady-state values of z and q are higher than in the case of accommodation (whenever accommodation is sustainable) as well as the in the case of unconstrained monopoly in both the leader's and the social planner's problem. As a numerical example, for the values used in Figure 2,[a] we obtain $z_{FB}(t) = 0.5261 - 0.1024\, e^{-0.1611\, t}$ and $q_{FB}(t) = 1.4706 - 0.4706\, e^{-0.1611\, t}$.

Now we analyze the second case in which the social planner solves the problem (FB) with an additional constraint that the leader's overall (net) profit needs to be non-negative (so called "constrained first-best"):

$$\int_0^{+\infty} \big[\big(A - p(t)\big)\big(p(t) - c_0(t)\big) - z^2(t) - F\big]e^{-rt}\,dt \geq 0. \qquad (IR)$$

It will be more useful to rewrite the problem (FB)–(IR) in terms of $q = A - p$ and $m = A - c_0$ (we also denote $\bar{m} = A - \bar{c}$):

$$I_{FB2} = \max_z \int_0^{+\infty} [q(t)(m(t) - q(t)) - z^2(t) - F + \tfrac{1}{2}q^2(t)]e^{-rt}\,dt, \qquad (FB2)$$

$$\text{subject to}$$

$$\dot{m}(t) = \mu[\bar{m} - m(t) + \sqrt{g}\,z(t)],$$

$$m(0) = \bar{m},$$

$$\int_0^{+\infty} [q(t)\big(m(t) - q(t)\big) - z^2(t) - F]e^{-rt}\,dt \geq 0.$$

[a]The values used there are $A = 5$, $c = 4$, $r = 0.05$, $g = 0.8$, $F = 0.0225$, and $\mu = 0.2$.

In this problem we have two control variables q and z and one state variable m.

We first argue that the last constraint in $(FB2)$ needs to be binding. If not, then the control variable q is "free" and in optimum we obtain $q = m$, which means marginal cost pricing. However, this clearly violates the last constraint due to the presence of fixed costs.

In order to solve the above problem, we form the Lagrangian which compared to the Hamiltonian contains an additional term for the last constraint, multiplied with a Lagrange multiplier ξ:

$$\begin{aligned}\mathcal{L} &= [q(m-q) - z^2 - F + \tfrac{1}{2}q^2]e^{-rt} + \eta\mu(\bar{m} - m + \sqrt{g}\,z) + \xi[q(m-q) - z^2 - F]e^{-rt} = \\ &= (1+\xi)[q(m-q) - z^2 - F]e^{-rt} + \tfrac{1}{2}q^2 e^{-rt} + \eta\mu(\bar{m} - m + \sqrt{g}\,z).\end{aligned}$$

Note that the Lagrange multiplier ξ is just a real number (i.e., it does not depend on time). Moreover, Kuhn-Tucker conditions require that $\xi \geq 0$. We obtain the following first-order conditions:

$$\begin{aligned}\mathcal{L}_z &= -2ze^{-rt}(1+\xi) + \mu\sqrt{g}\,\eta = 0, \\ \mathcal{L}_q &= (m - 2q)e^{-rt}(1+\xi) + qe^{-rt} = 0, \\ \mathcal{L}_m &= qe^{-rt}(1+\xi) - \eta\mu = -\dot{\eta}.\end{aligned}$$

This system can be solved in a similar way as the system for accommodation in the paper. First, we express η from the first condition and plug into the last one, to obtain a differential equation for z. Moreover, the second condition gives us a linear relationship $(1+\xi)m = (1+2\xi)q$. Thus we can eliminate q from the differential equation and obtain a system of two linear differential equations in z and m:

$$\begin{aligned}\dot{z} &= (r+\mu)z - \frac{1+\xi}{1+2\xi} \cdot \frac{\mu\sqrt{g}}{2}\, m, \\ \dot{m} &= \mu(\bar{m} - m + \sqrt{g}\,z).\end{aligned}$$

Note that this system still depends on the Lagrange multiplier ξ. We can solve it with ξ as parameter. The steady-state of the system is:

$$z^*_{FB2} = \frac{(1+\xi)\sqrt{g}}{2(1+2\xi)\rho - (1+\xi)g}\,\bar{m}, \qquad m^*_{FB2} = \frac{2(1+2\xi)\rho}{2(1+2\xi)\rho - (1+\xi)g}\,\bar{m}.$$

We can again proceed in the same way as in the paper. The full solution of the system is rather cumbersome and is not presented here.

Having the solution, in the next step we determine the value of the Lagrange multiplier ξ using the corresponding constraint, namely, the last constraint in $(FB2)$. However, the resulting equation cannot be solved analytically, as it involves a polynomial of degree 5. As a numerical example, for the values used in Figure 2 we obtain the Lagrange multiplier $\xi = 0.1795$ and the solution $z_{FB2}(t) = 0.4299 - 0.0716\, e^{-0.1667\,t}$, $m_{FB2}(t) = 1.3845 - 0.3845\, e^{-0.1667\,t}$, and $q_{FB2}(t) = 1.2017 - 0.3337\, e^{-0.1667\,t}$.

The patterns of z and q (for the above parameter values) are illustrated in the following figure.

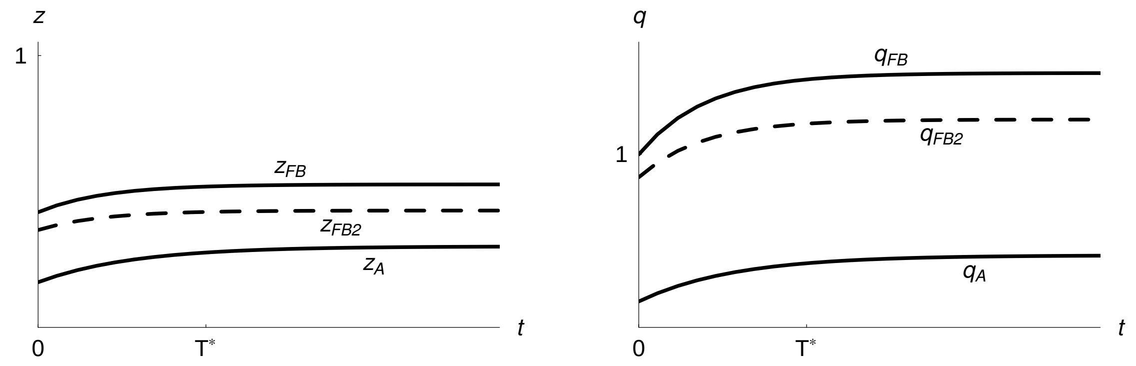

Figure 1: Patterns of z and q in accommodation and first-best