

International competition in vertically differentiated  
markets with innovation and imitation:  
Trade policy versus free trade

**Supplementary Appendix\***

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**B Appendix: Proofs**

*Derivation of Firm 2's cost function (4).* According to condition (i), without imitation, Firm 2's cost function has the form  $\frac{1}{2}\gamma_2 s_2^2$ , where  $\gamma_2$  is a constant such that  $\gamma_2 \geq \gamma_1$ . This yields marginal costs  $\gamma_2 s_2$ . According to (iii), these are also Firm 2's marginal costs when  $s_2 > s_1$ . On the other hand, when  $s_2 \leq s_1$ , the marginal costs can be, due to (ii), written as  $\gamma_2(1 - \mu)s_2$ , where  $\mu \in [0, 1]$  is a constant (parameter) representing the degree of imitation. After integration, we obtain that  $C_2(s_1, s_2) = C_2(s_1, 0) + \int_0^{s_2} \gamma_2(1 - \mu)\xi \, d\xi = \frac{1}{2}\gamma_2(1 - \mu)s_2^2$  when  $s_2 \leq s_1$ . Note that  $C_2(s_1, 0) = 0$  as for  $\mu = 0$  (no imitation), Firm 2's cost function needs to be equal to  $\frac{1}{2}\gamma_2 s_2^2$ , due to condition (i). Now, if  $s_2 > s_1$ , then due to continuity (condition (iv)), we obtain  $C_2(s_1, s_2) = C_2(s_1, s_1) + \int_{s_1}^{s_2} \gamma_2 \xi \, d\xi$ , which yields the expression given by (4).  $\square$

**Lemma 1.** *Firm 2's profit  $\pi_2(\sigma) - qc_2(\sigma)$  attains its maximum. The optimal value of  $\sigma$  lies in  $(0, 1) \cup (1, \infty)$ .*

*Proof of Lemma 1.* It can be directly verified that  $\pi_2''(\sigma) < 0$  for all  $\sigma > 0$ ,  $\sigma \neq 1$ . For an alternative argument, see Figure 1, which shows that in both regimes the derivative is decreasing on interval  $(0, 1)$  and on interval  $(1, \infty)$ , with a jump upwards at  $\sigma = 1$ . Hence, Firm 2's gross profit is concave on these intervals in both regimes. If  $\sigma \rightarrow \infty$ , it

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\*This Supplementary Appendix is available at: [www.uni-bonn.de/~kovac/papers/it](http://www.uni-bonn.de/~kovac/papers/it), as well as from the authors upon request.

can also be easily established that  $\pi_2(\sigma) - qc_2(\sigma) \rightarrow -\infty$  for any  $q > 0$ , since the costs are of a “higher order” than the gross profit. Thus, Firm 2’s (net) profit indeed attains a maximum.

Now we show that the maximum is not attained for  $\sigma \in \{0, 1\}$ . If  $\sigma = 1$  (minimal product differentiation), Firm 2’s net profit is negative (gross profit is zero, but the costs are positive). If  $\sigma = 0$ , the profit is zero. Moreover  $c'_2(0) = 0$ , which implies  $\pi'_2(0) - qc'_2(0) = \pi'_2(0) > 0$ . Therefore, Firm 2’s net profit is increasing and hence positive when  $\sigma$  is close to zero. This completes the proof.  $\square$

**Lemma 2.** *For all  $q \geq 0$ , the following statements hold:*

- (i) *There exists a unique solution of (13) on interval  $(0, 1)$ .*
- (ii) *There exists a solution of (13) on interval  $(1, \infty)$ , if and only if  $q < \bar{q}$ , where  $\bar{q} = \frac{4}{9}$  in the free trade regime, and  $\bar{q} = 1$  in the trade policy regime. This solution is then unique and does not depend on  $\mu$ .*

*Proof of Lemma 2.* (i) Since  $\pi'_2(0) - qc'_2(0) > 0 > \pi'_2(1) - qc'_2(1)$  for any  $q \geq 0$ , then (because of continuity) there always exists a solution of (13) on interval  $(0, 1)$ . Due to concavity, this solution is unique and represents the maximum of Firm 2’s net profit on  $(0, 1)$ .

(ii) If  $q \geq \frac{4}{9}$ , then  $\pi'_2(\sigma) < qc'_2(\sigma)$  for all  $\sigma > 1$  in the FT regime since the line  $qc'_2(\sigma)$  lies above the graph of  $\pi'_2(\sigma)$ ; see Figure 1. In this case  $\pi_2(\sigma) - qc_2(\sigma)$  is decreasing and, hence, is negative on  $[1, \infty)$ . On the other hand, if  $q < \frac{4}{9}$ , then  $\pi'_2(1) - qc'_2(1) > 0$ . Moreover,  $\pi'_2(\sigma) - qc'_2(\sigma) \rightarrow -\infty$  as  $\sigma \rightarrow \infty$  (see Table 2). Then (because of continuity), there always exists a solution of (13) on interval  $(1, \infty)$ . Due to concavity, this solution is unique and represents the maximum of Firm 2’s net profit on  $(1, \infty)$ .

The proof for the TP regime is analogous.  $\square$

**Lemma 3.** *Functions  $\sigma^1(q)$  and  $\sigma^2(q)$  (when defined) have a continuous first derivative (are  $\mathcal{C}^1$ ) and are decreasing in  $q$ .*

*Proof of Lemma 3.* Recall that by Lemma 2 the values  $\sigma^1(q)$  and  $\sigma^2(q)$  are (unique) solutions of (13) when  $\sigma < 1$  and  $\sigma > 1$ , respectively. Since both  $\pi'_2$  and  $c'_2$  are  $\mathcal{C}^1$ , it follows from the *Implicit Function Theorem* that the first derivative is continuous<sup>1</sup> and

$$[\pi''_2(\sigma^j(q)) - qc''_2(\sigma^j(q))] \frac{d\sigma^j(q)}{dq} = c'_2(\sigma^j(q)) \quad \text{for } j = 1, 2.$$

Since  $c'_2 > 0$ ,  $c''_2 > 0$ , and  $\pi''_2 < 0$ , we obtain  $d\sigma^j(q)/dq < 0$ , which completes the proof.  $\square$

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<sup>1</sup>Note that  $\frac{d}{ds}[\pi'_2(\sigma) - qc'_2(\sigma)] \neq 0$ , since  $c''_2 > 0$  and  $\pi''_2 < 0$ .

*Proof of Proposition 1.* Let us denote

$$\Delta(q) = [\pi_2(\sigma^2(q)) - qc_2(\sigma^2(q))] - [\pi_2(\sigma^1(q)) - qc_2(\sigma^1(q))],$$

which represents the difference between Firm 2's maximal net profits when it chooses  $\sigma < 1$  and when it chooses  $\sigma > 1$ . Therefore,  $\sigma^*(q) = \sigma^1(q)$  if  $\Delta(q) < 0$  and  $\sigma^*(q) = \sigma^2(q)$  if  $\Delta(q) > 0$ . We will show that  $\Delta(q)$  is decreasing and attains positive values when  $q \rightarrow 0^+$  and negative values when  $q \rightarrow \bar{q}^-$ . Since  $\Delta(q)$  is continuous, this would mean that there exists  $\hat{q} \in (0, \bar{q})$  such that  $\Delta(\hat{q}) = 0$ . Due to monotonicity,  $\Delta(q) > 0$  when  $q < \hat{q}$  and  $\Delta(q) < 0$  when  $q > \hat{q}$ . Clearly, the value of  $\hat{q}$  depends only on the parameter  $\mu$  (and not on  $\gamma_1$  and  $\gamma_2$ ).

First consider  $q \rightarrow \bar{q}^-$ . In this case,  $\sigma^2(q) \rightarrow 1$  and consequently  $\pi_2(\sigma^2(q)) - qc_2(\sigma^2(q)) < 0$ . Since the net profit for  $\sigma^1(q)$  is always positive (as optimal net profit when  $\sigma < 1$ ), then  $\Delta(q) < 0$  when  $q$  is close to  $\bar{q}$ . Note also that  $\sigma^1(q) \rightarrow 0$  when  $q \rightarrow \infty$  (see Figure 1).

Now consider  $q \rightarrow 0^+$ . In this case, we get  $\sigma^1(q) \rightarrow \frac{4}{7}$  in the FT regime and  $\sigma^1(q) \rightarrow \frac{2}{3}$  in the TP regime (see Table 2 and Figure 1). Consequently,  $\pi_2(\sigma^1(q)) - qc_2(\sigma^1(q))$  is bounded when  $q \rightarrow 0^+$ . In addition,  $\sigma^2(q) \rightarrow \infty$ . We show that then  $\pi_2(\sigma^2(q)) - qc_2(\sigma^2(q)) \rightarrow \infty$ . To see it, we use the first order condition (13), namely  $q\sigma^2(q) = \pi_2'(\sigma^2(q))$ , to obtain

$$\begin{aligned} \pi_2(\sigma^2(q)) - qc_2(\sigma^2(q)) &= \pi_2(\sigma^2(q)) - \frac{1}{2}[\sigma^2(q)]^2q + \mu q = \\ &= \pi_2(\sigma^2(q)) - \frac{1}{2}\sigma^2(q)\pi_2'(\sigma^2(q)) + \mu q. \end{aligned}$$

When  $q \rightarrow 0^+$ , the last term  $\mu q$  converges to zero and the rest to infinity since  $\pi_2(\sigma) - \frac{1}{2}\sigma\pi_2'(\sigma) \rightarrow \infty$  as  $\sigma \rightarrow \infty$ .<sup>2</sup> Hence,  $\Delta(q) \rightarrow \infty$  when  $q \rightarrow 0^+$ .

As the last step, we show that  $\Delta(q)$  is decreasing. Now recall that  $\sigma^1(q)$  and  $\sigma^2(q)$  represent local maxima of Firm 2's net profit  $\pi_2(\sigma) - qc_2(\sigma)$ . Thus, it follows from the *Envelope Theorem* that

$$\frac{d\Delta(q)}{dq} = -c_2(\sigma^2(q)) + c_2(\sigma^1(q)).$$

This is negative since  $c_2(q)$  is increasing and by definition  $\sigma^1(q) < 1 < \sigma^2(q)$ .  $\square$

*Proof of Proposition 2.* (ii) We start with the second part. The statement follows from the *Envelope Theorem*. The derivative of Firm 2's optimal profit with respect to  $\mu$  is

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<sup>2</sup>It can be easily established that  $\pi_2(\sigma) - \frac{1}{2}\sigma\pi_2'(\sigma)$  is equal to  $2\sigma^3(4\sigma - 7)/(4\sigma - 1)^3$  in the FT regime and to  $9\sigma^3(2\sigma - 3)(2\sigma - 1)(12\sigma^2 - 13\sigma + 4)/[2(4\sigma - 1)^3(3\sigma - 2)^3]$  in the TP regime. Both these expressions diverge to infinity when  $\sigma \rightarrow \infty$ .

equal to  $\frac{1}{2}q[\sigma^1(q)]^2$  when  $q > \hat{q}_\mu$ , and to  $\frac{1}{2}q$  when  $q < \hat{q}_\mu$ . Obviously, it is positive in both cases (when  $q \neq 0$ ).

(i) The claim that  $\sigma^*(q)$  is increasing in  $\mu$  (for a fixed  $q$ ) is simply a consequence of other statements.

First, the proof that  $\sigma_\mu^1(q)$  is increasing in  $\mu$  is analogous to the proof that  $\sigma_\mu^1(q)$  is decreasing in  $q$  since  $\sigma_\mu^1(q)$  depends only on the product  $q \cdot (1 - \mu)$ ; see the Proof of Lemma 3.

Second, in order to establish that  $\sigma^2(q)$  does not depend on  $\mu$ , it is sufficient to recognize that the derivatives of Firm 2's gross profits and costs also do not depend on the parameter  $\mu$ ; see the Proof of Lemma 2.

Third, let  $\hat{\sigma}_\mu^j = \sigma^j(\hat{q}_\mu)$ , for  $j = 1, 2$ . The values of  $\hat{\sigma}_\mu^1$ ,  $\hat{\sigma}_\mu^2$ , and  $\hat{q}_\mu$  are determined simultaneously by the system of three equations introduced before the proposition. According to the last equation,

$$\pi_2(\hat{\sigma}_\mu^1) - \frac{1}{2}(1 - \mu)\hat{q}_\mu(\hat{\sigma}_\mu^1)^2 = \pi_2(\hat{\sigma}_\mu^2) - \frac{1}{2}\hat{q}_\mu[(\hat{\sigma}_\mu^2)^2 - \mu].$$

Taking the derivative with respect to  $\mu$  (and using the *Implicit Function Theorem*), we obtain

$$\begin{aligned} [\pi_2'(\hat{\sigma}_\mu^1) - (1 - \mu)\hat{q}_\mu\hat{\sigma}_\mu^1] \frac{d\hat{\sigma}_\mu^1}{d\mu} + \frac{1}{2}\hat{q}_\mu(\hat{\sigma}_\mu^1)^2 - \frac{1}{2}(1 - \mu)(\hat{\sigma}_\mu^1)^2 \frac{d\hat{q}_\mu}{d\mu} = \\ = [\pi_2'(\hat{\sigma}_\mu^2) - \hat{q}_\mu\hat{\sigma}_\mu^2] \frac{d\hat{\sigma}_\mu^2}{d\mu} + \frac{1}{2}\hat{q}_\mu - \frac{1}{2}[(\hat{\sigma}_\mu^2)^2 - \mu] \frac{d\hat{q}_\mu}{d\mu}. \end{aligned}$$

According to the first-order conditions (first and second equation), the first term on the left-hand side and the first term on the right-hand side are equal to zero. Therefore,

$$[(\hat{\sigma}_\mu^2)^2 - \mu - (1 - \mu)(\hat{\sigma}_\mu^1)^2] \frac{d\hat{q}_\mu}{d\mu} = \hat{q}_\mu[1 - (\hat{\sigma}_\mu^1)^2],$$

which means that  $d\hat{q}_\mu/d\mu > 0$  since  $(\hat{\sigma}_\mu^2)^2 - \mu - (1 - \mu)(\hat{\sigma}_\mu^1)^2 = 2[c_2(\hat{\sigma}_\mu^2) - c_2(\hat{\sigma}_\mu^1)] > 0$  (as  $c_2$  is increasing) and  $\hat{\sigma}_\mu^1 < 1$ .  $\square$

**Lemma 4.** For any  $\alpha \in (0, 1]$  and  $\mu \in [0, 1]$ , Firm 1's profit  $q\pi_1(\sigma^*(q)) - \frac{1}{2}\alpha q^2$  attains its maximum. The optimal value of  $q$  is positive.

*Proof of Lemma 4.* The leader's profit is clearly continuous for all  $q \neq \hat{q}$ . We show that the leader's profit (i) diverges to  $-\infty$  when  $q \rightarrow \infty$ ; (ii) converges to 0 when  $q \rightarrow 0^+$ ; and (iii) is increasing in  $q$  when  $q$  is sufficiently small.

(i) If  $q \rightarrow \infty$ , then  $q > \hat{q}_\mu$  and  $\sigma^*(q) = \sigma^1(q)$ . We have argued in the proof of Proposition 1 that  $\sigma^*(q) \rightarrow 0^+$ . In addition,  $\pi_1(\sigma)$  is bounded in some neighborhood of

$\sigma = 0$ , as it converges to a finite limit when  $\sigma \rightarrow 0^+$  (see Table 2). Thus,  $q\pi_1(\sigma) - \frac{1}{2}\alpha q^2$  diverges to  $-\infty$ .

(ii) If  $q \rightarrow 0^+$ , then  $q < \hat{q}_\mu$  and  $\sigma^*(q) = \sigma^2(q)$ . Similarly, we have shown in the proof of Proposition 1 that  $\sigma^*(q) \rightarrow \infty$ . Again,  $\pi_1(\sigma)$  is bounded when  $\sigma$  is sufficiently large, as it converges to a finite positive limit (which is  $\frac{1}{16}$  in the FT regime and  $\frac{1}{144}$  in the TP regime), when  $\sigma \rightarrow \infty$  (see Table 2). Hence,  $q\pi_1(\sigma) - \frac{1}{2}\alpha q^2$  converges to zero.

(iii) The derivative of Firm 1's continuation net profit with respect to  $q$  is

$$\pi_1(\sigma^*(q)) + q\pi_1'(\sigma^*(q))\frac{d\sigma^*(q)}{dq} - \alpha q. \quad (\text{B.1})$$

We show that this derivative is positive when  $q \rightarrow 0^+$ . Recall that for a sufficiently small  $q$  (namely  $q < \hat{q}_\mu$ ), we get  $\sigma^*(q) = \sigma^2(q)$ , which is continuously differentiable.

Now consider  $q \rightarrow 0^+$ . Then  $\sigma^*(q) \rightarrow \infty$ . Thus,  $\pi_1(\sigma)$ , which is the first term in (B.1), converges to a positive limit, as shown in claim (ii). Obviously, the last term converges to zero. We will show now that  $A = \pi_1'(\sigma^*(q)) \cdot d\sigma^*(q)/dq$  has a finite limit, which implies that the second term (equal to  $qA$ ) also converges to zero when  $q \rightarrow 0^+$ . For this recall that the first-order condition for Firm 2's profit maximization is  $\pi_2'(\sigma) = q\sigma$ . In addition, it follows from the proof of Lemma 3 that

$$\frac{d\sigma^*(q)}{dq} = \frac{\sigma}{\pi_2''(\sigma) - q} = \frac{\sigma^2}{\sigma\pi_2''(\sigma) - \pi_2'(\sigma)}.$$

We may, therefore, rewrite  $A$  in terms of  $\sigma$  and take the limit  $\sigma \rightarrow \infty$ . We obtain  $A = \sigma^2\pi_1'(\sigma)/[\sigma\pi_2''(\sigma) - \pi_2'(\sigma)]$ , which after computing the derivatives (from Table 2) implies that in the FT and the TP regime

$$A^{\text{FT}} = -\frac{(2\sigma + 1)(4\sigma - 1)}{4(16\sigma^2 - 16\sigma + 21)},$$

$$A^{\text{TP}} = -\frac{(\sigma - 1)^2(2\sigma - 1)(3\sigma - 2)(4\sigma - 1)(7\sigma + 2)}{9(576\sigma^6 - 2112\sigma^5 + 3684\sigma^4 - 3504\sigma^3 + 1909\sigma^2 - 568\sigma + 72)}.$$

Thus,  $A^{\text{FT}}$  converges to  $-\frac{1}{8}$  and  $A^{\text{TP}}$  converges to  $-\frac{7}{216}$ , when  $\sigma \rightarrow \infty$ . This completes the proof.  $\square$

*Proof of Proposition 3.* (i) The *Envelope Theorem* implies that the (total) derivative of Firm 1's optimal profit is equal to

$$\frac{\partial}{\partial \mu}[q\pi_1(\sigma_\mu^1(q)) - \frac{1}{2}\alpha q^2] = q\pi_1'(\sigma_\mu^1(q))\frac{\partial \sigma_\mu^1(q)}{\partial \mu},$$

evaluated at the optimal value of  $q$ . It follows from part (i) of Proposition 2 that

$\partial\sigma_\mu^1(q)/\partial\mu > 0$ . In addition, it can be easily established that  $\pi_1'(\sigma) < 0$  when  $\sigma < 1$  (see the expressions in Table 2). This completes the proof.

(ii) Here we proceed as suggested in Footnote 28 by expressing  $q = \pi_2'(\sigma)/c_2'(\sigma)$  from the follower's best response (13). As we now consider only the case where the leader produces a higher quality, i.e.,  $\sigma < 1$ , then  $c_2'(\sigma) = (1 - \mu)\sigma$  and

$$(1 - \mu)q = \frac{\pi_2'(\sigma)}{\sigma} \quad (\text{B.2})$$

After substituting this into the leader's profit, we obtain a maximization problem with respect to a single variable  $\sigma$ . Taking the derivative we obtain the first order condition

$$\frac{\alpha}{1 - \mu} = \frac{\sigma\pi_1(\sigma)}{\pi_2'(\sigma)} + \frac{\sigma^2\pi_1'(\sigma)}{\sigma\pi_2''(\sigma) - \pi_2'(\sigma)}. \quad (\text{B.3})$$

By solving the system of equations (B.2) and (B.3) we would obtain the candidate for equilibrium when the leader produces a higher quality. This system, however, involves polynomials of higher order, so it is not possible to find the solution explicitly. However, we can still derive the comparative statics with respect to  $\mu$  by plotting the graphs of the functions. We first use (B.3) to determine the effect of  $\mu$  on the equilibrium value of  $\sigma$ . Then we use the condition

$$\alpha q = \pi_1(\sigma) + \frac{\sigma\pi_1'(\sigma)\pi_2'(\sigma)}{\sigma\pi_2''(\sigma) - \pi_2'(\sigma)}, \quad (\text{B.4})$$

obtained by multiplication of (B.2) and (B.3), to determine the equilibrium relation between  $\sigma$  and  $q$ ; see Figure E.1. We can see from the figures that the right-hand side of (B.3) is increasing in  $\sigma$  in both FT and TP regimes. Moreover, the right-hand side of (B.4) is increasing in  $\sigma$  in the FT regime for all  $\sigma$ , and in the TP regime for  $\sigma > 0.0633$ .

Summing up, if  $\mu$  increases, then  $\alpha/(1 - \mu)$  increases. Therefore,  $\sigma^*$  increases as well, and  $\alpha q^*$  decreases. This proves that  $q^*$  is decreasing in  $\mu$ . This statement holds for all  $\mu$  in the FT regime and when  $\alpha/(1 - \mu) > 0.0649$  in the TP regime (where 0.0649 is the value that the right-hand side of (B.3) reaches for  $\sigma = 0.0633$ ).

(iii) The claim follows directly from two facts. First, Firm 1's (net) profit does not depend on  $\mu$  directly. Second, according to part (i) of Proposition 2, Firm 2's best response does not depend on  $\mu$ , whenever it leads to a higher quality, i.e., when  $\sigma^*(q) > 1$ .

(iv) The claim follows directly from the *Envelope Theorem* and from the fact that Firm 2's maximization problem (for a fixed  $q$ ) and, hence, its best response function  $\sigma^*$  do not depend on  $\alpha$ .  $\square$

## C Appendix: Additional comparisons

In this appendix we describe results from additional comparisons that are omitted in the paper.

**Firms' profits.** As noted in the paper (Section 6), Firm 1's profit is harmed in the TP regime due to profit shifting. For all  $\alpha \in (0, 1]$  and  $\mu \in [0, 1]$ , Firm 1 earns a lower profit (both net and gross) in the TP equilibrium than in the FT equilibrium. This relationship is, however, reversed for Firm 2 in most of the parameter space. With the exception of a small region with area approximately 1%, its profit (both net and gross) in the FT equilibrium is lower than the (corresponding) profit in the TP equilibrium; see Figures E.3 and E.4.

**Prices and qualities.** In order to obtain some intuition behind the comparisons of the profits (see above as well as Section 6 in the paper), we compare firms' prices, qualities, and hedonic prices. Consistently with our intuition, it is indicative that trade policy reduces Firm 1's quality, but increases Firm 2's quality. Consequently, Firm 1 sets a higher price in the FT equilibrium (than in the TP equilibrium). On the other hand, Firm 2 sets a lower price in the FT equilibrium. These results hold always (for all  $\alpha, \mu \in [0, 1]$ ) for Firm 1 and almost always (with exception of small regions with areas approximately 6%, as illustrated on Figures E.6 and E.5) for Firm 2. On the other hand, the comparison of hedonic prices (price-quality ratios) does not yield such clear-cut predictions. Firm 1's hedonic price in the FT equilibrium is higher than in the TP equilibrium in approximately 21% of cases, whereas for Firm 2 the share is higher in approximately 7% of cases, as shown on Figures E.7 and E.8. In addition, if trade policy reduces Firm 2's hedonic price, it also reduces Firm 1's hedonic price.<sup>3</sup>

**Quality ratios.** Given the comparison of firms' qualities, the above results suggest (and simulations confirm) that the quality ratio  $\sigma = s_2/s_1$  in the FT equilibrium is always lower than the quality ratio in the TP equilibrium. However, this comparison does not provide much of informative value, since  $\sigma < 1$  in the FT equilibrium and  $\sigma > 1$  in the TP equilibrium when quality reversal occurs.

A better measure for comparing the qualities available on the market is the quality gap  $\max\{\sigma, 1/\sigma\}$ . A lower value of the quality gap (closer to 1) is connected with less differentiation and, in some sense, to a tougher competition.<sup>4</sup> The simulations show that the optimal trade policy increases the quality gap if and only if quality reversal occurs.

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<sup>3</sup>In particular, the shaded area in Figure E.8 is one component of the shaded area in Figure E.7.

<sup>4</sup>Note, however, that a simultaneous increase in both firms' qualities may benefit both firms as well as consumers, but it will leave the quality gap unchanged.

Hence, optimal trade policy leads to a softer competition when it leads to quality reversal. On the other hand, it leads to a tougher competition, when no quality reversal occurs.

## D Appendix: Corner solutions

In this appendix we provide more details about so-called “corner solutions” that occur when  $q_\mu^* = \hat{q}_\mu$ . In this case, Firm 2 is indifferent between choosing a lower and a higher quality than Firm 1. As we already mentioned in the paper (Footnote 31), as a byproduct of the simulations, we find that at the point of discontinuity  $\hat{q}$ , Firm 1’s net profit jumps upwards. Thus, only the situation when Firm 2 chooses  $\sigma^1(q)$  can occur in equilibrium (recall that we analyze pure strategy equilibria).<sup>5</sup> Figure E.9 shows an example of Firm 1’s continuation profit in such a case.

The simulations further show that corner solution occurs in both FT and TP regimes (in equilibrium). In particular, in the FT regime it occurs when both  $\alpha$  and  $\mu$  are sufficiently large (the red region on Figure E.10), and in the TP regime it occurs near the boundary of quality reversal (the blue region on Figure E.10). The former is particularly of our interest, because it is related to the kink in the comparison of domestic welfare in Figure 6 in the paper (see also Footnote 37). More specifically, this kink lies in the intersection between the region of the corner solution (red region on Figure E.10) and the region where free trade yields a higher domestic welfare (shown in Figure 6 in the paper), as indicated on Figure E.11. As we mentioned in the paper (Footnote 35), the kink occurs for  $\alpha \approx 0.84$  and  $\mu \approx 0.82$  and the boundary is downward-sloping when  $\alpha$  increases. This is indicated in Table E.1.

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<sup>5</sup>Otherwise, a slight increase in  $q$  would increase Firm 1’s profit.

## E Appendix: Figures and tables

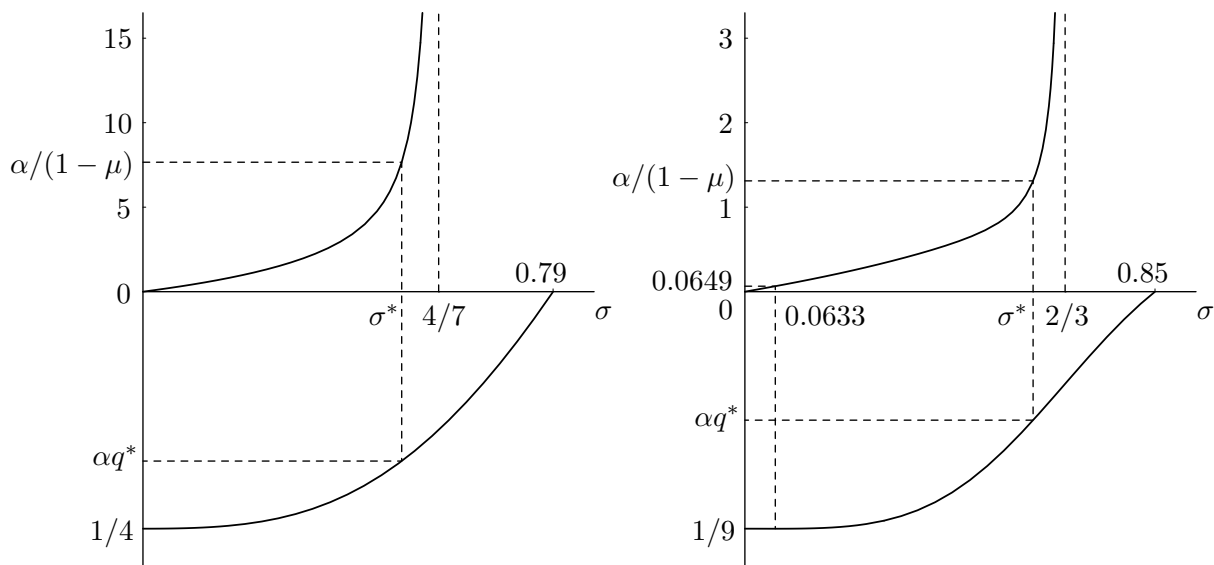


Figure E.1: Comparative statics with respect to  $\mu$  in the FT equilibrium (left) and the TP equilibrium (right). The upper curve represents the function on the right-hand side of (B.3), the lower curve the function on the right-hand side of (B.4) turned upside down.

*Note.* Figures E.2–E.8 show the comparisons of some variables in the FT equilibrium and in the TP equilibrium. The shaded area corresponds to the values of parameters  $\alpha$  and  $\mu$  where the equilibrium value of the variable in the FT equilibrium is higher than in the TP equilibrium. The black curve represents the borderline between Firm 1 choosing higher quality in both regimes and quality reversal (see Figure 4 from the paper).

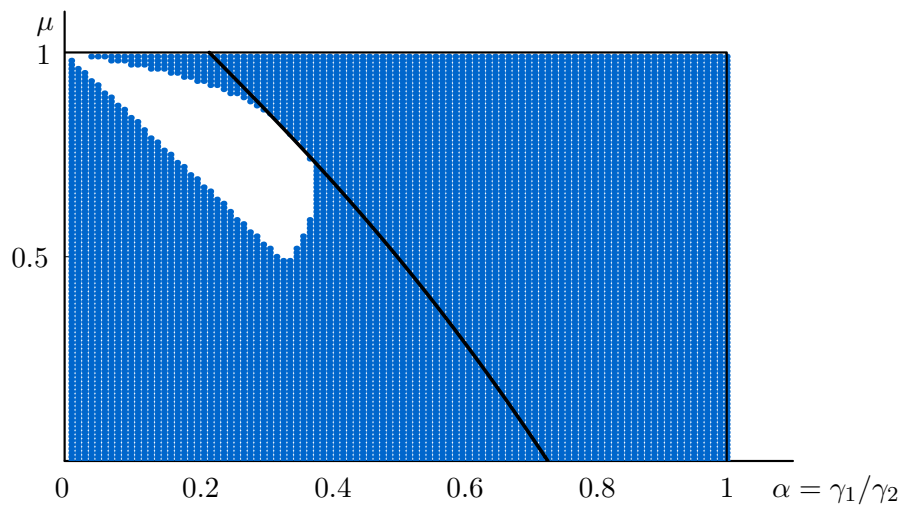


Figure E.2: Comparison of the market size (num. simulations)

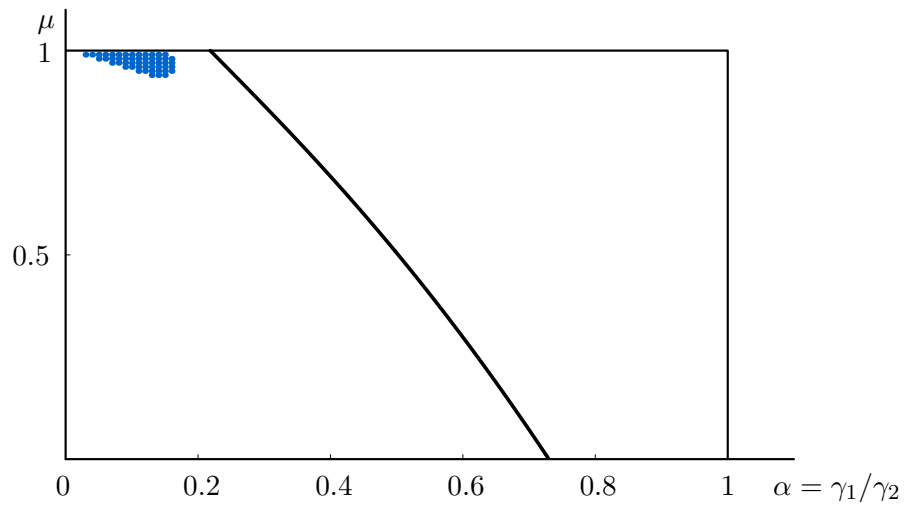


Figure E.3: Comparison of Firm 2's net profit (num. simulations)

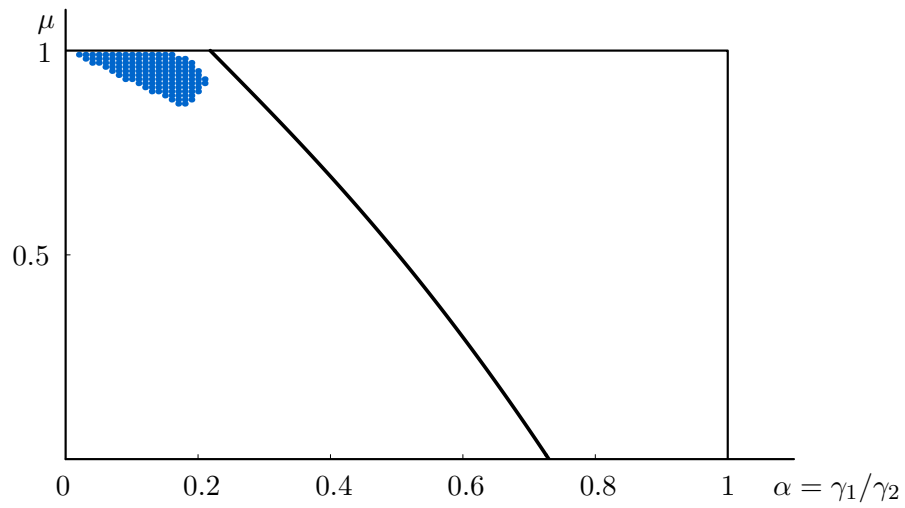


Figure E.4: Comparison of Firm 2's gross profit (num. simulations)

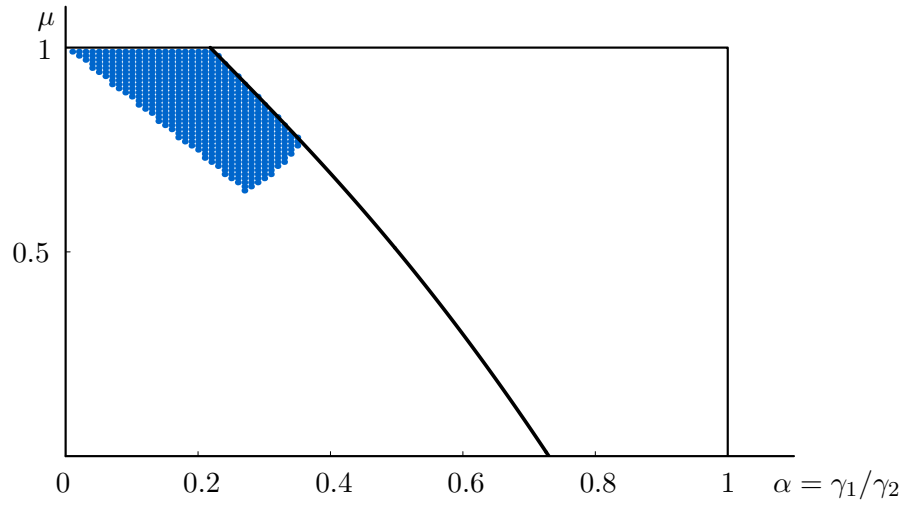


Figure E.5: Comparison of Firm 2's price  $p_2$  (num. simulations)

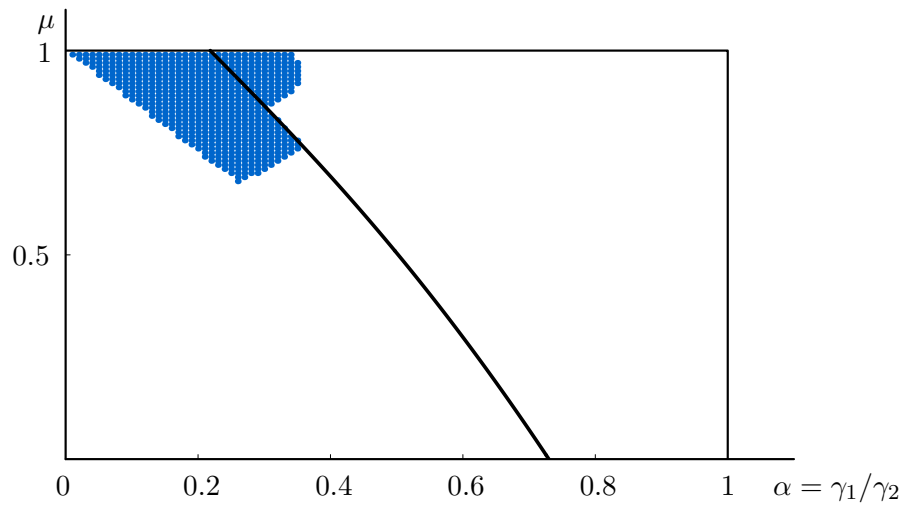


Figure E.6: Comparison of Firm 2's quality  $s_2$  (num. simulations)

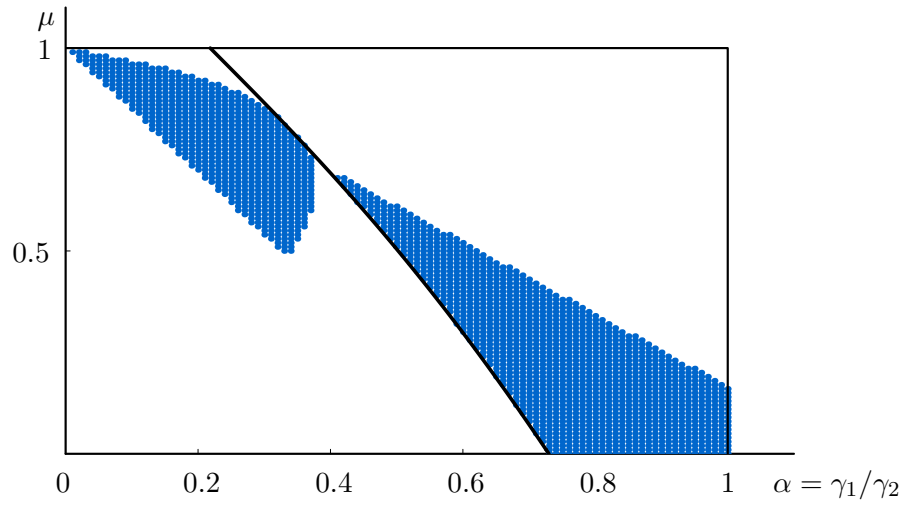


Figure E.7: Comparison of Firm 1's hedonic price  $p_1/s_1$  (num. simulations)

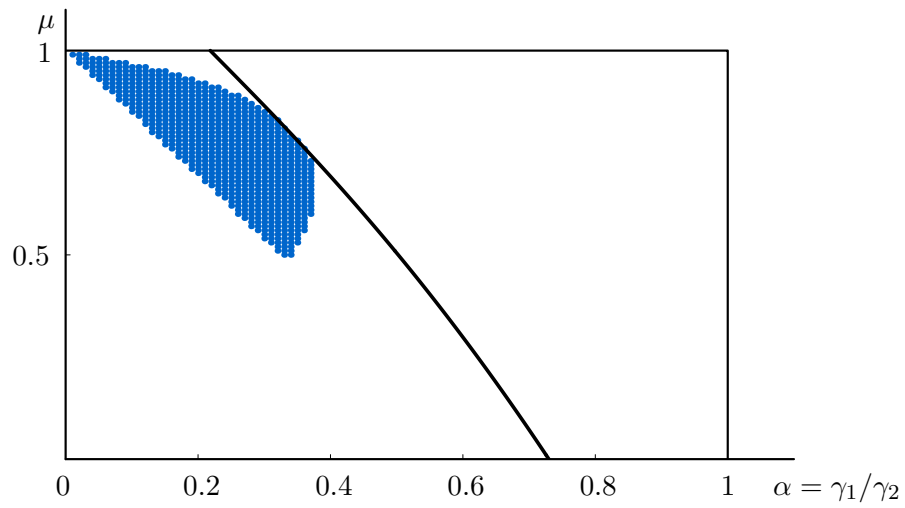


Figure E.8: Comparison of Firm 2's hedonic price  $p_2/s_2$  (num. simulations)

	$\alpha = 0.9$	$\alpha = 1$
FT	0.063462	0.063462
	$\wedge$	$\vee$
TP	0.063506	0.063409

Table E.1: Numerical values of domestic welfare for  $\mu = 0.842$

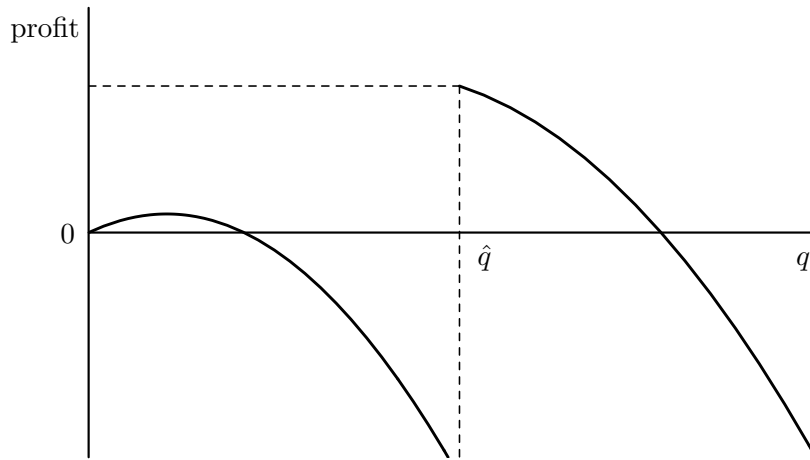


Figure E.9: Leader's net (continuation) profit  $q\pi_1(\sigma^*(q)) - \frac{1}{2}\alpha q^2$  with corner solution (numerical example: FT regime,  $\alpha = \frac{9}{10}$ ,  $\mu = \frac{9}{10}$ )

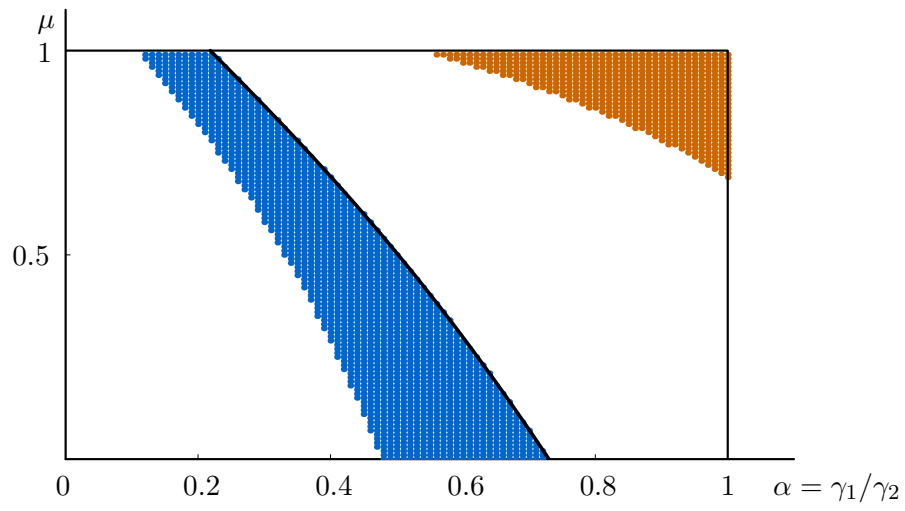


Figure E.10: Regions with corner solutions (num. simulations)

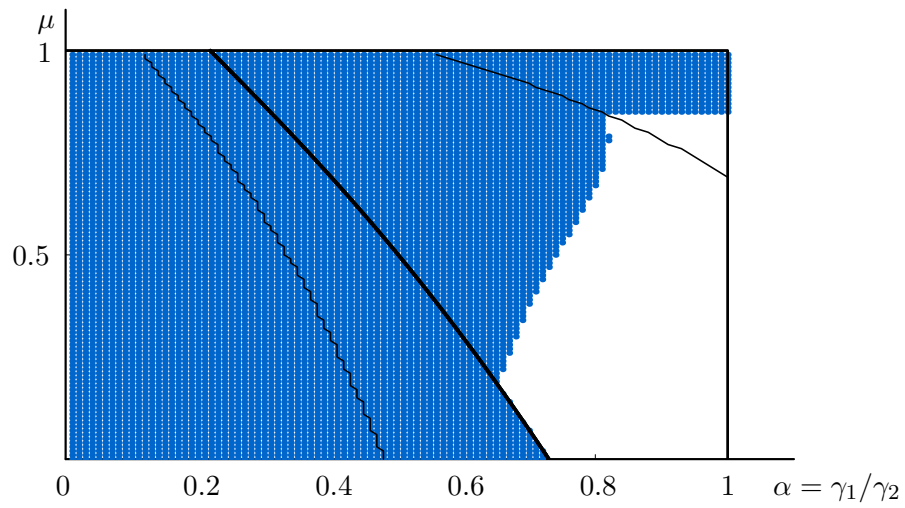


Figure E.11: Domestic welfare and corner solutions (num. simulations)