Product bundling and tying in competition*

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Abstract

This paper explores tying in the situation where a multi-product firm without monopoly power competes against several single-product firms. It considers two independent markets: one for a horizontally differentiated good, the other for a homogeneous good. As opposed to the widely accepted opinion that tying may be profitable only in the case of monopoly power, it is shown that under reasonable assumptions tying is profitable for the multi-product firm even without monopoly power, and it has a negative welfare effect. Further, in certain cases all firms may benefit from bundling.

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JEL classification: L13, L11, L41

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1 Introduction

Tying refers to the situation where a firm makes the purchase of one of its products conditional on the purchase of another of its products. According to the leverage theory, tying "provides a mechanism whereby a firm with monopoly power in one market can use the leverage provided by this power to foreclose sales in, and thereby monopolize, a second market" (Whinston 1990). Therefore, tying is one of the basic concepts in anti-trust laws and policies. Particular cases deal with firms that try to monopolize another market[1] and require proof of the monopoly power in the first market. This proof is often omitted in practice "since how could a tie-in be imposed unless such power existed?" (Posner 1976, p. 172), suggesting that in general tying may be profitable only when a monopoly is present.

The theoretical literature on tying focuses either on the monopolization of the second market (following the above idea) or on tying of complementary components of a system (e.g., hardware and software) produced by several multi-product firms. However, the effect of tying by a multi-product firm facing specialized competitors is not properly addressed yet. According to the argument by Posner (1976), the multi-product firm will never find tying profitable. However, casual evidence indicates that tying in such a situation also occurs in the marketplace.[2] In this paper, I analyze the situation where a multi-product firm (without monopoly power) competes against several single-product firms to show that tying can indeed be a reasonable strategy for the multi-product firm, but it has a negative effect on welfare. This finding is important for assessing the relevance of anti-trust laws and policies.

I introduce a model in which one multi-product (generalist) firm competes in two markets for two non-complementary goods. The first good is heterogeneous and is produced by another (specialist) firm. The second good is homogeneous,[3] and I consider various structures of the market for it, in particular duopoly and perfect competition. If the generalist firm decides to offer its products only as a bundle,[4] two effects emerge: a *competition softening effect* and a *substitution effect*. The former means that the competition on the market for the second good becomes softer and a mark-up is added to its price, which has a positive impact

[1] Recall, for example, the famous case of the U.S. vs. Microsoft.

[2] Microsoft's tying of Word, Excel, Access, PowerPoint and other programs into Office can serve as an example. See Denicolo (2000) for more details.

[3] These assumptions are satisfied, for example, in the well known Czech anti-trust case S21/95-240 of Likérka STOCK Plzeň-Božkov, which was tying several alcoholic beverages to Fernet Stock. Fernet Stock was considered horizontally differentiated from other bitter liqueurs (e.g., Becherovka, Jägermeister,...) whereas the other alcoholic beverages were considered homogeneous products.

[4] Bundling is a more general concept than tying and refers to the situation when a package containing at least two different products is offered. The practice in which the firm offers only the bundle is called pure bundling, as opposed to mixed bundling when the firm also offers some of the products separately.

on the generalist firm's incentives to bundle. The latter means that consumers with low valuation for the second good may switch from the bundle offered by the generalist firm and buy the products separately. This has usually a negative impact on the generalist firm's incentives to bundle. The profitability of bundling depends on the interaction of those two effects.

In particular, I identify two cases where the generalist firm prefers pure bundling. In the first case, the competition softening effect is significant and the substitution effect is weak. This occurs, for example, when there is duopoly on the market for the second good and the first good is differentiated sufficiently. The most striking is the second case, where the substitution effect is very strong, so that it subsequently causes a structural change on the first market.[5] The structural change means that the market becomes more segmented which increases firms' market power over certain groups of consumers. As a consequence, the prices rise making all firms better off, however, at the expense of consumers and overall welfare. This result suggests that anti-trust authorities need to be aware of bundling cases where no rival is damaged, and hence, no firm will appeal to them.

The theoretical literature does not properly address tying by a multi-product firm without monopoly power competing against several single-product firms. However, there is literature partially related to this problem. This literature can be divided into two streams: literature on the abuse of tying to create or preserve market power, in particular a monopoly (leverage theory), and literature on bundling of compatible products produced by several multi-product firms.

After heavy criticism of the leverage hypothesis in the 1950s and the 1970s (so-called "Chicago critique"),[6] Whinston (1990) reconsiders it with a theoretical model. Using a simple model where a multi-product firm with monopoly on one market competes in price with a rival on another market, he examines the implications of tying and comes to the conclusion that it may lead to the foreclosure of the monopolist's rival in the tied good market. However, the monopolist will engage in tying only if it can commit itself to doing so, which will consequently drive its rival out of the market. Whinston (1990) claims that tying is profitable for the monopolist precisely because of the "exclusionary effect on the market structure." The welfare effects in his model are, however, uncertain.

Besides foreclosure, several other effects of tying by a firm with monopolistic power in one market were identified. Carbajo, de Meza and Seidmann (1990) argue that under imperfect competition (e.g., duopoly) on the tied good market, bundling may cause rivals to compete less aggressively. Seidmann (1991) claims that bundling may enable rivals to avoid competition for tied good sales. Hence, tying has a favorable effect on a rival that does not tie, which contradicts the common interpretation of the leverage theory. He also applies his analysis to

[5] This occurs in situations where the degree of differentiation of the first good is low and valuations for the second good are heterogeneous enough.

[6] See Director and Levi (1956) and Posner (1976).

several anti-trust cases to demonstrate the relevance of his study. Carlton and Waldman (2002) argue that a dominant firm can use bundling to remain dominant in an industry with rapid technological change. They apply this analysis to the Microsoft case and claim that Microsoft's tying and deterrence of Netscape's entry into the market for Internet browsers could have increased social welfare.

The second stream of literature focuses on bundling of compatible complementary products produced by several multi-product firms (also called mix-and-match literature). Matutes and Regibeau (1988) consider two firms, each of them producing two necessary components of a system, with horizontally differentiated components produced by different firms. Later, Matutes and Regibeau (1992) introduce bundling in their former model and show that in many cases the firms choose to produce compatible components but offer discounts to consumers who buy the components from one firm. They argue that bundling makes the competition tougher, since a price cut raises sales of both products yielding stronger incentives to undercut the rival.

Current literature knows only two papers which discuss tying by a multi-product firm without monopoly power which competes against several single-product firms: Denicolo (2000) and Nalebuff (2000). Both analyze a framework similar to Matutes and Regibeau (1992), i.e., horizontally differentiated markets and complementary products, with specialized competitors instead of one multi-product firm. Denicolo (2000) considers two horizontally differentiated markets and concludes that the necessary condition for bundling to be profitable for the multi-product firm is that the markets are sufficiently asymmetrically differentiated. In this case, the bundle is more differentiated than the less differentiated product, which benefits the multi-product firm and its rival on the less differentiated market. On the other hand, Nalebuff (2000) considers a larger number of symmetrically differentiated markets and concludes that bundling is profitable for the multi-product firm, only if the size of the bundle is sufficiently high.[7]

The models presented in this paper differ crucially from Denicolo (2000) and Nalebuff (2000) in several points. First, I do not require complementarity of the products and consider only one differentiated product whereas the other is homogeneous. Contrary to Denicolo (2000) and Nalebuff (2000), I also identify a case with structural changes where all firms are better off than in the case of no bundling. Second, I also analyze mixed bundling which cannot be interpreted in Denicolo's the framework of incompatibility and compatibility. Third, I also focus on the welfare effects of bundling which are only briefly discussed by both authors.

The remainder of the paper is organized as follows. In Section 2, I present the basic model to illustrate the competition softening effect. This model is completely tractable and allows for a complete classification of the generalist firm's decision. In Section 3, I extend the basic model by assuming heterogeneous but discrete valuations for the second good which allow for the substitution effect. This model

[7]In the model, at least four products need to be bundled.

is also tractable and it generalizes the results of the basic model. In Section 4, I assume continuous distribution of valuations for the second good to check the robustness of the results from previous sections. In Section 5, I conclude, discussing the relevance of my results for anti-trust policies, and suggest possible extensions. Appendix A contains the proofs of all lemmas and propositions.

2 Basic model

I start with a simple model which illustrates the competition softening effect. There are two markets for indivisible goods X_1 and X_2 and three firms: firm G operating on both markets and firms A and B, each of them operating only on the market for good X_1 and X_2, respectively. Production of good X_i, $i \in \{1, 2\}$ involves a constant unit cost $c_i \geq 0$, which is the same for every firm producing it. Goods X_1 produced by firms G and A are horizontally differentiated (X_{G1} and X_{A1} denote their version of good X_1)[8]; good X_2 is homogeneous. To model the differentiation, I use a Hotelling model with firms positioned on the edges of the unit interval: firm G at 0 and firm A at 1.

Consumers are indexed by two parameters α and v. The marginal utility for the first unit of good X_1 purchased from a firm positioned at x_1 is $w - \theta|\alpha - x_1|$; the marginal utility from the first unit of good X_2 is v. The marginal utility from any additional unit of each good is zero. With these assumptions, each consumer will buy either one unit of a good or he will not buy it at all. In this basic model, I assume that v is constant ($v > c_2$) and α is uniformly distributed over the interval $[0, 1]$.

The expression $\theta|\alpha - x_1|$ represents the transportation costs (assumed to be linear) or equivalently the disutility from getting a differentiated product. The parameter θ stands for the unit transportation costs or the degree of differentiation of products X_{A1} and X_{G1}. For simplicity let w be high enough to have the market for good X_1 covered in equilibrium (either as a separate product or in a bundle). Note that the same effect is achieved if I simply assume that every consumer wants to buy good X_1. Under the above assumptions about marginal utility, the utility function of the consumer indexed by parameters α and v is additive and takes the form

$$u_{\alpha,v}(h, x_1, x_2) = h + w - \theta|\alpha - x_1| + x_2 v, \tag{1}$$

where x_1 is the position of the firm from which the consumer purchases good X_1; x_2 is the quantity of good X_2 he purchases ($x_2 \in \{0, 1\}$); and h is the amount of money spent on all other goods. I assume wealth is identical for all consumers (denoted by m) and is high enough to purchase any combination of goods X_1 and X_2 available in equilibrium. Note that the assumption of identical wealth is not

[8] I have also investigated the case of vertically differentiated products. Because the results are similar to the results in this section, they are not presented here.

restrictive because of additivity of the utility function, the consumer's decision does not depend on his level of wealth (provided m is high enough).

The whole situation can be modelled as a two-stage game. In the first stage, firm G decides which combination of goods X_1 and X_2 it will sell — its options are listed in Table 1. In the second stage, all firms compete in prices.[9] I analyze the pure-strategy equilibria of each subgame and look for a subgame perfect equilibrium of the whole game.

Strategy	Products offered
no bundling	X_{G1} and X_2
pure bundling	bundle $\mathcal{G} = \{X_{G1}, X_2\}$
mixed bundling	$\mathcal{G}$ and X_{G1} $\mathcal{G}$ and X_2 $\mathcal{G}$, X_{G1} and X_2

Table 1: Strategies of firm G in the first stage

One assumption requires more attention. I assume that firm G can precommit itself not to change its bundling strategy in the second stage (e.g., not to sell one of the goods separately if it previously decided otherwise). This assumption is rather common in the literature in bundling and is used by many authors, e.g., Whinston (1990) and Choi and Stefanadis (2001). In particular, Whinston (1990) suggests that this precommitment can be achieved, for example, by technological settings or product design, which may involve sunk costs. This can also include commitment to mixed bundling. For example, Microsoft's Windows Media Player (WMP) was for a long period (until recently) bundled with OS Windows. Hence, playing certain audio and video files (`.wma` and `.wmv`) was impossible (or at least very difficult) under other operating systems like Linux or Mac OS.[10] This can be interpreted as mixed bundling, since it was possible to use other competing applications under Windows.[11]

When the competition softening effect is present, without the commitment assumption, firm G has incentives to introduce the product X_2 in order to achieve positive profit in the market for good X_2. This consequently wipes out the substitution effect. The analysis of the mixed bundling case where firm G offers the bundle as well as both goods X_{G1} and X_2 suggests that the equilibrium in a game without commitment would be the same as in the no bundling subgame.

[9] When comparing some outcomes (e.g., profits, prices, etc.) of those subgames, I always mean the equilibrium outcomes.

[10] See, for example, `www.pcquest.com/content/linux/handson/103010201.asp` or `en.wikipedia.org/wiki/Windows_Media_Player`.

[11] In addition, due to the European Commission antitrust ruling, Microsoft was forced to produce a Windows XP version without WMP called Windows XP N, which in the end is very unsuccessful; see, for example, `news.com.com/2100-1016_3-5960750.html`.

Remark 1. To avoid unintuitive cases I usually, with one exception, assume that consumers who are indifferent among several choices decide randomly among them.[12] The exception is the situation where the consumer is indifferent between buying a good or not buying it at all. In this case, I assume that he chooses the former.

Remark 2. In addition, I assume that if a firm cannot earn a positive profit, its best response is to set the price equal to marginal cost. This avoids equilibria where some firm's profit has a maximum of zero, the firm is indifferent among several prices yielding a zero profit (one of them being marginal costs), and it chooses a price above marginal costs. The assumption restricts the analysis to such equilibria where no firm earns negative profit, and if a firm earns zero profit, its price is equal to marginal costs. Further, with the claim that a subgame has no equilibrium in pure strategies, I always mean that it does not have any equilibrium satisfying the above condition (see in particular Proposition 5 and Lemma 5).

In order to be able to judge welfare implications, I compare consumer surplus and total surplus across subgames. The former is defined as

$$CS = \int_0^1 u^*_{\alpha,v} \, \mathrm{d}\alpha,$$

where $u^*_{\alpha,v}$ denotes the equilibrium utility of consumer α. The latter is defined as the sum of consumer surplus and total industry profits.

2.1 No bundling

Consider first the benchmark case where firm G decides to sell its products separately. Let p_{j1} be the price of good X_1 offered by firm j (where $j = G, A$). Similarly, let p_{j2} be the price of good X_2 offered by firm j (where $j = G, B$) and denote $p_2 = \min\{p_{G2}, p_{B2}\}$. In the absence of bundling, the consumer can choose among four options. Table 2 shows his utility from each choice (note that good X_2 can be purchased by any firm offering it and that the rest of the wealth is spent on other goods). The additivity of the utility function implies that the decisions to buy goods X_1 and X_2 are independent.[13]

Bertrand competition on the market for good X_2 yields the equilibrium prices $p_{G2} = p_{B2} = c_2$, at which every consumer is going to buy it. In that case, the profits from selling good X_2 are $\Pi_{G2} = \Pi_B = 0$. The Hotelling competition on the market for good X_1 implies that in equilibrium, the market is split equally (consumers with $\alpha < \frac{1}{2}$ buy X_{G1} and consumers with $\alpha > \frac{1}{2}$ buy X_{A1}) and

$$p_{G1} = p_{A1} = \theta + c_1, \qquad \Pi_{G1} = \Pi_A = \frac{1}{2}\theta; \tag{2}$$

[12] All the results also hold with the weaker assumption that if the measure of indifferent consumers is positive, then for every choice there is a positive measure of consumers choosing it.

[13] Whinston (1990) refers to such a situation as an independent pricing game.

Combination	Utility
X_{G1}	$m + w - p_{G1} - \theta\alpha$
X_{A1}	$m + w - p_{A1} - \theta(1 - \alpha)$
X_{G1} and X_2	$m + w - p_{G1} - \theta\alpha - p_2 + v$
X_{A1} and X_2	$m + w - p_{A1} - \theta(1 - \alpha) - p_2 + v$

Table 2: Consumer's options in the absence of bundling

see, for example, Shy (1996), pp. 149–152 for detailed computations. Therefore, the profit of firm G is $\Pi_G = \Pi_{G1} + \Pi_{G2} = \frac{1}{2}\theta$ and the total industry profit is θ. For the prices given by (2), the consumer surplus can be computed as

$$\begin{aligned} CS &= \int_0^{1/2} \big(m + w - (\theta + c_1) - \theta\alpha - c_2 + v\big)\, \mathrm{d}\alpha + \\ &+ \int_{1/2}^{1} \big(m + w - (\theta + c_1) - \theta(1 - \alpha) - c_2 + v\big)\, \mathrm{d}\alpha = \\ &= m + w + v - c_1 - c_2 - \tfrac{5}{4}\theta, \end{aligned}$$

which yields the total surplus $TS = m + w + v - c_1 - c_2 - \frac{1}{4}\theta$.

2.2 Pure bundling by firm G

In the case of pure bundling, the consumer has only three options (assuming that he needs to buy good X_1): good X_{A1} alone; both goods X_{A1} and X_2 (from firm B); and the bundle $\mathcal{G} = \{X_{G1}, X_2\}$; he cannot buy X_{G1} separately. The bundle is offered at the price p_G and yields the utility $m + w - p_G - \theta\alpha + v$ (the utility in the other cases is the same as in Table 2). If $p_{B2} < v$, the combination X_{A1}, X_2 is strictly preferred to X_{A1} by all consumers. If $p_{B2} = v$, all consumers who are indifferent between buying and not buying X_2 in addition to X_{A1} choose by assumption to buy it (see Remark 1). Therefore, for $p_{B2} \leq v$, each consumer decides only between buying both X_{A1} and X_2, and buying the bundle $\mathcal{G} = \{X_{G1}, X_2\}$. On the other hand, if $p_{B2} > v$, the consumer decides between buying X_{A1} and buying the bundle $\mathcal{G}$. Given the prices p_{A1}, p_{B2}, p_G, the consumer indexed by parameters α and v buys the bundle if and only if $\alpha \leq \alpha^*(v)$, where $\alpha^*(v)$ corresponds to an indifferent consumer. Obviously, $\alpha^*(v)$ is given by the equation $v - p_G - \theta\alpha = v - p_{A1} - p_{B2} - \theta(1-\alpha)$ if $p_{B2} \leq v$, and by the equation $v - p_G - \theta\alpha = -p_{A1} - \theta(1-\alpha)$ if $p_{B2} > v$. Hence,

$$\alpha^*(v) = \frac{p_{A1} + \min\{v, p_{B2}\} - p_G}{2\theta} + \frac{1}{2}. \tag{3}$$

In the rest of this section, I will drop the argument v since it is assumed to be identical for all consumers.

Obviously, any $p_{B2} > v$ is dominated. The above discussion implies that for $p_{B2} \le v$, all consumers decide only between buying both X_{A1} and X_2, or buying the bundle. Thus, there is no substitution effect in this model. Note that this result is crucially based on the fact that all consumers have identical valuation for good X_2.

If $0 \le \alpha^* \le 1$ and $p_{B2} \le v$, the profits of the firms are

$$\begin{aligned} \Pi_G &= \tfrac{1}{2\theta}(p_G - c_1 - c_2)(\ \ p_{A1} + p_{B2} - p_G + \theta), && (4)\\ \Pi_A &= \tfrac{1}{2\theta}(p_{A1} - c_1 \qquad)(-p_{A1} - p_{B2} + p_G + \theta), && (5)\\ \Pi_B &= \tfrac{1}{2\theta}(p_{B2} \qquad - c_2)(-p_{A1} - p_{B2} + p_G + \theta). && (6) \end{aligned}$$

The equilibrium of this subgame is characterized in the following lemmas (their proofs can be found in Appendix A).

Lemma 1. *If $v - c_2 \ge \frac{3}{4}\theta$, the equilibrium prices of the pure bundling subgame are*

$$p_G = \frac{5}{4}\theta + c_1 + c_2, \qquad p_{A1} = \frac{3}{4}\theta + c_1, \qquad p_{B2} = \frac{3}{4}\theta + c_2. \tag{7}$$

Remark 3. This result is consistent with Denicolo (2000), who also considers the case when one of the products is less differentiated in Lemma 2. He receives the same prices as above for $\theta = 1$ and $c_1 = c_2 = 0$.

In the previous lemma, I assumed $v - c_2 \ge \frac{3}{4}\theta$ in order to ensure that $p_{B2} \le v$ in equilibrium. If this assumption is violated, firm B sets the price $p_{B2} = v$ (I assume that indifferent consumers decide to buy good X_2; see Remark 1) leading to another equilibrium which is characterized in the following lemma.

Lemma 2. *If $v - c_2 < \frac{3}{4}\theta$, the equilibrium prices of the pure bundling subgame are*

$$p_G = \theta + c_1 + \frac{v + 2c_2}{3}, \qquad p_{A1} = \theta + c_1 - \frac{v - c_2}{3}, \qquad p_{B2} = v. \tag{8}$$

In order to simplify the analysis, I introduce a new parameter,

$$\mu = \min\left\{\tfrac{1}{4}\theta,\ \tfrac{1}{3}(v - c_2)\right\}, \tag{9}$$

which allows me to write the equilibrium prices in one expression and to obtain the following proposition. Note that $\mu > 0$; its interpretation follows.

Proposition 1. *The equilibrium prices of the pure bundling subgame are*

$$p_G = \theta + c_1 + c_2 + \mu, \qquad p_{A1} = \theta + c_1 - \mu, \qquad p_{B2} = c_2 + 3\mu, \tag{10}$$

yielding firm G's market share $\frac{1}{2} + \frac{1}{2\theta}\mu$, profits

$$\Pi_G = \frac{1}{2\theta}(\theta + \mu)^2, \quad \Pi_A = \frac{1}{2\theta}(\theta - \mu)^2, \quad \Pi_B = \frac{3}{2\theta}\mu(\theta - \mu), \tag{11}$$

and consumer surplus

$$CS = m + w + v - c_1 - c_2 - \tfrac{5}{4}\theta - \tfrac{3}{2}\mu + \tfrac{1}{4\theta}\mu^2.$$

Compared to the case of no bundling, the price of good X_2 is higher by 3μ (which is obviously positive). The price of the bundle is higher by μ than the sum of prices of X_{G1} and X_2 in the case of no bundling, but the price of X_{A1} is lower by μ. The intuition behind this has been set out previously in the introduction. When firm G withdraws from the market for good X_2, the competition there becomes softer (competition softening effect), in which case firm B is the only seller of standalone good X_2. However, there is no monopoly because the consumer may switch from good X_2 to the bundle. Nevertheless, the price of good X_2 can rise by 3μ and (due to strategic complementarity) the price of the bundle may rise by μ. Therefore μ can be interpreted as the markup which is added to the price of the bundle in the case of pure bundling due to the competition softening effect. However, the effect on the price of good X_{A1} is inverse because every consumer buys either the bundle or both X_{A1} and X_2 (which is more expensive), so firm A has to decrease its price not to lose too many customers when B increases its price. Obviously, there is no substitution effect because in equilibrium all consumers buy good X_2.

As for the profits, obviously firm G earns a higher profit than do firms A and B together. Comparing them to the case of no bundling, the profit of firm A is lower, but the profits of firms B and G are higher. The total industry profit $\theta + \frac{3}{2}\mu - \frac{1}{2\theta}\mu^2$ is obviously higher than in the case of no bundling. Using the above proposition, the total surplus is

$$TS = m + w + v - c_1 - c_2 - \tfrac{1}{4}\theta - \tfrac{1}{4\theta}\mu^2,$$

which is lower than in the case of no bundling. I conclude that firm G always prefers pure bundling to selling separate products, which has a negative effect on firm A, but a positive effect on firm B. In addition, this practice decreases both consumer surplus and total surplus.

Remark 4. Note that the difference between the profits vanishes as the degree of differentiation θ approaches zero. Then, the goods become perfect substitutes, the markup μ converges to zero, and pure bundling yields in equilibrium the same profits as selling separate products.

Remark 5. In order to facilitate comparison to the literature, in particular, Matutes and Regibeau (1992), I briefly address the case where firms A and B would be integrated (and sell the products separately). Behavior of such an integrated firm (denoted as AB) crucially differs from behavior of specialized firms, even if it does not bundle. In this case, provided that $p_{B2} > c_2$, the consumers decide between buying the bundle from firm G for the price p_G, and buying both products separately from firm AB. Compared to the non-integrated case with two specialized firms, the competition softening effect does not occur in full magnitude since any increase in p_{B2} decreases also the demand for integrated firm's good X_{A1}. It can be easily computed that $p_G = \theta + c_1 + c_2$ and $p_{A1} + p_{B2} = \theta + c_1 + c_2$ in equilibrium, yielding the same profits and utilities as in the no bundling subgame.

2.3 Mixed bundling by firm G

Mixed bundling means that besides offering the bundle firm G, also offers good X_{G1} or X_2 or both. I will analyze each of these three cases separately.

2.3.1 Firm G offers the bundle and good X_{G1}

Consider first the situation where firm G decides to offer the bundle as well as good X_{G1}. In this case, the pair of goods X_{G1} and X_2 is a perfect substitute for the bundle (this can be seen from the utility function), and to choose between them the consumers simply compare p_G and $p_{G1} + p_{B2}$. Therefore, by also selling good X_{G1} firm G "competes against itself." This means that it may be willing to charge a very high price if good X_{G1} is purchased separately. The following proposition confirms this intuition and characterizes the equilibrium.

Proposition 2. *In the subgame where firm G offers the product X_{G1} as well as the bundle, the equilibrium prices of goods X_{A1}, X_{B2}, and the bundle are the same as in the pure bundling subgame (see Proposition 1) and X_{G1} may have any price p_{G1} such that*

$$p_{G1} > \theta + c_1 - \frac{1}{2\theta}\mu(\theta - 3\mu). \tag{12}$$

In this case, no consumer buys X_{G1} together with X_{B2}, and the equilibrium profits are the same as in the pure bundling subgame.

If the above condition does not hold, firm B can decrease its price slightly below $p_G - p_{G1}$, in which case all consumers purchase good X_2; see Appendix A for computations.

Obviously, the above equilibrium is outcome equivalent to the equilibrium in the pure bundling subgame.[14] Condition (12) means that firm G sells good X_{G1} for a very high price so that it does not compete with the bundle. However, since $\theta > 3\mu$, this price may still be lower than the equilibrium price of X_{G1} in the no bundling subgame, given by (2).

At this point I should stress the relevance of the above results for anti-trust policies. If firm G is not allowed to engage in pure bundling, it may achieve the same outcome also by mixed bundling when offering the good X_{G1} for a very low price (lower than in the no bundling case) but high enough so that no one buys it separately (note that the price of good X_2 is higher).

2.3.2 Firm G offers the bundle and good X_{G2}

If firm G offers the bundle as well as good X_2 (for the price p_{G2}), Bertrand competition on the market for good X_2 yields $p_{G2} = p_{B2} = c_2$ in equilibrium. Therefore,

[14]Two equilibria are *outcome equivalent* if they yield the same utility to each consumer and the same profit to each firm.

the equilibrium in this case is analogous to the case where the market for good X_2 is perfectly competitive. In this case, after firm G's withdrawal from the market for good X_2, the price of good X_2 will remain equal to c_2 (the same occurs also in the case when there are at least three firms selling good X_2).

Obviously, there is no competition softening effect in both cases described above. Intuitively, it implies that there cannot be any markup added to the price of good G. Hence, $p_G = \theta + c_1 + c_2$ and $p_{A1} = \theta + c_1$ in equilibrium, which is outcome equivalent to the case of no bundling. The following proposition clearly confirms the intuition.

Proposition 3. *In the subgame where firm G offers the bundle and good X_2, the equilibrium prices are*

$$p_G = \theta + c_1 + c_2, \qquad p_{A1} = \theta + c_1, \qquad p_{G2} = p_{B2} = c_2, \tag{13}$$

yielding firm G's market share $\frac{1}{2}$ and the same profits as in the no bundling subgame.

Corollary 1. *If the market for good X_2 is perfectly competitive, the equilibrium prices of the pure bundling subgame are given by* (13)*. Firm G is indifferent between selling separate products and pure bundling.*

Remark 6. Note that by taking $\mu = 0$ in (10) and (11), I obtain exactly the same prices and profits as in Proposition 3. This corresponds either to the situation when X_{G1} and X_{A1} are perfect substitutes (i.e., $\theta = 0$) or to a situation when the marginal cost of good X_2 is very high (i.e., $c_2 = v$). Obviously, in both of these cases bundling does not pay; see also Remark 4.

2.3.3 Firm G offers the bundle and both goods X_{G1} and X_{G2}

The last case to consider is where firm G offers the bundle as well as goods X_{G1} and X_2 separately. Using the same arguments as above, $p_{G2} = p_{B2} = c_2$. The consumers' decision whether to buy the bundle or goods X_{G1} and X_2 (either from B or G) is determined by their prices p_G and $p_{G1} + c_2$. Obviously, there is again no competition softening effect in this case. Therefore, $\min\{p_G, p_{G1} + c_2\} = \theta + c_1 + c_2$ and $p_{A1} = \theta + c_1$ in equilibrium. Every such equilibrium is outcome equivalent to the case of no bundling.

2.4 Subgame-perfect equilibrium

The above analysis of the pricing subgames implies that firm G prefers pure bundling over other marketing strategies.[15] However, both consumer surplus and total surplus are lower than in the no bundling subgame.

[15] The same outcome can also be achieved by mixed bundling when firm G also sells good X_{G1}, but for such a price that nobody wants to buy it.

3 Heterogeneous valuations of good X_2

In this section, I extend the basic model to illustrate the substitution effect and to investigate when it arises. For this purpose, I consider heterogeneous valuations for good X_2, starting with a simple discrete case with only two types of consumers: with a low valuation v_L and a high valuation v_H. In the next section, I analyze the case where the valuation of good X_2 is distributed uniformly over the interval $[0, 1]$. Furthermore, I restrict the analysis to comparing the subgame where firm G sells separate products as a benchmark with the pure bundling subgame and the mixed bundling subgame where firm G sells product X_2 as well as the bundle (because of its interpretation that the market for good X_2 is perfectly competitive; see the discussion preceding Proposition 3 and Corollary 1).

Remark 7. Other types of mixed bundling, in particular when firm G offers good X_{G1} as well as the bundle, may also have interesting implications. Introduction of good X_{G1} (at price p_{G1} such that $p_{G1} < p_G < p_{G1} + c_2$) can serve for firm G as a tool for price discrimination between consumers with high and low valuation for good X_2. Obviously, this weakens the substitution effect. Hence, a specific form of mixed bundling may be preferred by firm G to both selling separate products and pure bundling. However, a detailed analysis of these cases significantly extends the discussion and is beyond the scope of this paper.

I assume that for each $v \in \{v_L, v_H\}$, the parameter α is distributed uniformly on $[0, 1]$, independently from valuation v, and there is an equal measure of consumers with valuations v_L and v_H. In other words, the consumers are distributed uniformly on the set $[0, 1] \times \{v_L, v_H\}$, with density $\frac{1}{2}$.[16] In order to illustrate the substitution effect, I assume that $v_L < c_2 < v_H$, which means that the consumers with low valuation for good X_2 will not buy it separately. In addition, I assume that $v_L = 0$ and $v_H = 1$ (then $0 < c_2 < 1$). This assumption significantly simplifies the analysis and does not weaken the results.[17]

3.1 No bundling

When firm G sells its product separately, consumers' decision which type of good X_1 to buy is the same as in the previous model, yielding the same equilibrium prices and profits as in (2). Analogically, Bertrand competition on the market for good X_2 implies that $p_{G2} = p_{B2} = c_2$ in equilibrium, yielding zero profits. Hence,

[16] The model can be generalized by assuming that the measure of consumers with valuations v_H and v_L are λ and $1 - \lambda$, where $\lambda \in [0, 1]$. For $\lambda \to 1$, this model reduces to the basic model analyzed in the previous section. However, the discussion of such model becomes much more complicated.

[17] Because of the additivity of the utility function and the particular type of the distribution, any condition on the parameters can be written as a linear inequality in $v_H - c_2$, $c_2 - v_L$, and θ. Hence, my choice of v_H and v_L is in principle only a normalization $(v_H - c_2) + (c_2 - v_L) = 1$. Note that also in the basic model μ depends only on $v - c_2$.

consumers with $v = 1$ buy good X_2, and consumers with $v = 0$ do not. The consumer surplus in this case is $CS = m + w - c_1 + \frac{1}{2}(1 - c_2) - \frac{5}{4}\theta$; the total surplus is $TS = m + w - c_1 + \frac{1}{2}(1 - c_2) - \frac{1}{4}\theta$.

3.2 Pure bundling

Firm G's decision to bundle causes the substitution effect. This means that some consumers who otherwise prefer to buy X_1 from firm G now also obtain good X_2 in the bundle, and hence, may switch to firm A. Therefore, the substitution effect should decrease firm G's incentives to bundle. However, this effect may be so significant that none of the consumers with valuation 0 for good X_2 buy the bundle, i.e., $\alpha^*(0) \leq 0$, where α^* is defined by (3). In this case, the effect on firm G is inverse since it gives firm A market power over consumers with low valuation for good X_2. Hence, firm A (and consequently firm G) may relax its price.

Depending on the sign of $\alpha^*(0) = \frac{1}{2\theta}(\theta + p_{A1} - p_G)$, I will consider three cases:

$$\text{(a) } \alpha^*(0) > 0, \qquad \text{(b) } \alpha^*(0) < 0, \qquad \text{(c) } \alpha^*(0) = 0.$$

Based on this classification, I will call an equilibrium *type* (a) *equilibrium*, if $\alpha^*(0) > 0$ in this equilibrium. Analogically, I define *type* (b) *equilibrium* and *type* (c) *equilibrium.*

An easy computation yields the market share of firm G:

$$q_G = \begin{cases} \frac{1}{2\theta}(\theta + p_{A1} + \frac{1}{2}p_{B2} - p_G), & \text{if } \alpha^*(0) \geq 0, \\ \frac{1}{4\theta}(\theta + p_{A1} + \ \ p_{B2} - p_G), & \text{if } \alpha^*(0) \leq 0. \end{cases}$$

Then the market share of firm A is $q_{A1} = 1 - q_G$, and the market share of firm B is $q_{B2} = \frac{1}{4\theta}(\theta - p_{A1} - p_{B2} + p_G)$. Note that each firm A's and G's profit functions are concave on both sets where $\alpha^*(0) \geq 0$ or $\alpha^*(0) \leq 0$. However, they may still have two local maxima. Taking the first derivative of firm G's profit, I obtain

$$\frac{\partial \Pi_G}{\partial p_G} = \begin{cases} \frac{1}{2\theta}(\theta + p_{A1} + \frac{1}{2}p_{B2} - 2p_G + c_2), & \text{if } \alpha^*(0) > 0, \\ \frac{1}{4\theta}(\theta + p_{A1} + \ \ p_{B2} - 2p_G + c_2), & \text{if } \alpha^*(0) < 0. \end{cases} \tag{14}$$

Depending on the sign of $\alpha^*(0)$ and the value of p_{B2} (interior or one of two corner solutions), there can be nine types of equilibria. The following lemma claims that there is no equilibrium where $\alpha^*(0) = 0$.

Lemma 3. *There is no type* (c) *equilibrium in the pure bundling subgame.*

The equilibria in the remaining six cases are classified in Table 3 in Appendix B. The table contains the differences between the price and marginal costs, profits, and the characterization of the bundling decision of firm G. All the computations are straightforward, and I omit them. The third column of the table specifies necessary conditions for $\alpha^*(0)$ to have the correct sign and for p_{B2} to have the appropriate value, i.e., condition $c_2 < p_{B2} < 1$ in cases (a1) and (b1), condition $\partial\Pi_B/\partial p_{B2}|_{p_{B2}=1} \geq 0$ in cases (a2) and (b2), and whether $\partial\Pi_B/\partial p_{B2}|_{p_{B2}=c_2} \leq 0$ in cases (a3) and (b3).

In addition to these conditions, it is necessary to check that none of the firms prefers to switch to the case where $\alpha^*(0)$ has the opposite sign. For example, consider prices given in Table 3 in Appendix B for case (a1). Firm G may prefer to raise its price above $p_{A1} + \theta = \frac{19}{10}\theta + c_1 + \frac{1}{5}c_2$ (see Table 3), i.e., to achieve the case where $\alpha^*(0) < 0$. Firm G's profit has another local maximum in $(p_{A1} + \theta, \infty)$ if and only if $\partial\Pi_G/\partial p_G|_{p_G=p_{A1}+\theta}$ is positive from right or equivalently $\theta < \frac{16}{13}c_2$. In this case, for $p_G = \frac{5}{4}\theta + c_1 + c_2$, firm G's profit attains its maximum $\frac{25}{64}\theta$ in the interval $(p_{A1} + \theta, \infty)$, which is higher than the original profit if and only if $\theta < \frac{4}{343}(44 + 25\sqrt{2})c_2 \approx 0.9245\, c_2$. Hence, it is necessary to exclude all cases where the last inequality holds. Analogical computations need to be performed for all six cases for firms A and G (firm B cannot affect $\alpha^*(0)$). The last column of the table contains the results (called the "no switch condition"). Obviously, these together with the necessary conditions from the third column are sufficient and necessary for the existence of each type of equilibrium. According to these conditions, Figure 1 in Appendix B classifies the equilibria by parameters c_2 and θ. The thin dashed lines show the region which is excluded because of the conditions that no firm prefers to switch.

The figure shows that there is a region where none of the equilibria exists. Hence, there is no equilibrium in pure strategies of the pure bundling subgame. This fact is caused by non-concavity of the profit functions, which may have more local maxima. Therefore, a firm may prefer to change its price drastically to achieve another maximum causing a discontinuity in the reaction curves (see also Remark 2). In particular, as discussed above, firm G may prefer to switch to a case where $\alpha^*(0)$ has the opposite sign.

The following proposition summarizes the results about firm G's bundling decision in cases where an equilibrium in pure strategies exists.

Proposition 4. *Provided that there exists an equilibrium in pure strategies, firm G prefers pure bundling if and only if either of the following conditions holds:*

(i) $c_2 < \frac{1}{2}$ *and* $\theta > 2c_2$,

(ii) $\theta < 4(1 - c_2)$ *and* $\theta \leq \frac{4}{337}(27 + 14\sqrt{2})c_2$,

(iii) $4(1 - c_2) \leq \theta \leq \frac{1}{5}c_2 + \frac{1}{20}(2 + 3\sqrt{2})$.

The profits of firms A and G satisfy the inequality $\Pi_G > \frac{1}{2}\theta > \Pi_A$ in case (i)*, and $\Pi_A > \Pi_G > \frac{1}{2}\theta$ in cases* (ii) *and* (iii).

Condition (i) corresponds to the region above the thick dashed curve in Figure 1 and clearly confirms the intuition about the competition softening effect and the substitution effect. It means that c_2 must be low enough so that the competition on the market for good X_2 can be softened sufficiently, i.e., there is a significant competition softening effect, and θ must be high enough so that it is not easy for consumers with low valuation for good X_2 to switch from the bundle to good X_{A1}, i.e., there is a weak substitution effect. The inequalities $\Pi_G > \frac{1}{2}\theta > \Pi_A$ mean that whenever pure bundling is preferred in case (a) it makes firm A worse off, which is a similar result as in the basic model.

The most interesting case occurs when conditions (ii) and (iii) hold. These correspond to equilibria of types (b1) and (b2), respectively. Then the substitution effect is very strong and has an inverse effect on firm G. If $\alpha^*(0) < 0$, all consumers with valuation 0 for good X_2 buy only product X_{A1} from firm A. This way firm G commits not to serve the consumers with valuation 0 for good X_2, and only the consumers with valuation 1 are relevant for competition between firms A and G. Therefore, firm A has a market power over consumers with a low valuation for good X_2, and any increase in the price of good X_1 reduces the demand by less than it would have done in case (a).[18] In other words, the market for good X_1 becomes segmented, with one segment being served only by firm A. This changes the structure of the competition and allows firm A, and hence, also firm G to relax their prices, which makes pure bundling profitable. As opposed to the case where condition (i) holds, here I get $\Pi_A > \Pi_G > \frac{1}{2}\theta$, which means that firm A earns higher profit than firm G, and all firms are better off than in the case of no bundling.

However, the effect on both consumer surplus and total surplus is negative. In case (b2), the consumer surplus and total surplus are $CS = m + w - c_1 + \frac{1}{2}(1 - c_2) - \frac{319}{128}\theta$ and $TS = m + w - c_1 + \frac{1}{2}(1 - c_2) - \frac{57}{128}\theta$, respectively. Both are obviously lower than in the case of no bundling. Comparison in cases (a1), (a2), and (b2) also leads to the conclusion that both consumer surplus and total surplus are lower than in the case of no bundling. However, the formulas are more complicated, and they do not provide direct insight. All computations are straightforward, and I omit them.

The above result is particularly important in the light of competition policy. It shows that tying may make all firms better off whereby the effect on welfare is negative. Hence, no firm is harmed by tying and has incentives to start a case against the generalist firm. This suggests that the anti-trust authorities need to be more active in identifying bundling cases with negative welfare effects.

Remark 8. Observe that in a type (a) equilibrium, there are consumers who purchase the bundle containing the good X_B with no value for them. If unbundling is possible and a secondary market is available, one should expect those consumers to resell this good. If, however, the firms anticipate this, they will change their

[18] In the classical Hotelling model (with one good), a similar effect is achieved if firm A is shifted to the point $\frac{1}{2}$, and the degree of differentiation is doubled. This yields prices $p_{G1} = \frac{5}{3}\theta$, $p_{A1} = \frac{7}{3}\theta$ and profits $\Pi_{G1} = \frac{25}{36}\theta$ and $\Pi_A = \frac{49}{36}\theta$. Not surprisingly, they are the same as in case (b3) where $p_{B2} = c_2$.

strategies yielding to another equilibrium. This represents an interesting variation of my model, but is beyond the scope of this paper.

3.3 Mixed bundling by firm G

If firm G sells product X_2 as well as the bundle, Bertrand competition on the market for good X_2 implies that its price is equal to c_2. Hence, all prices and profits are the same as specified in Table 3 in Appendix B in cases (a3) and (b3). The only difference is that now the condition $\partial\Pi_B/\partial p_{B2}|_{p_{B2}=c_2} \leq 0$ is not required to hold. The following proposition summarizes the relevant conditions.

Proposition 5. *In the subgame where firm G sells the bundle as well as good X_2, the following statements hold:*

(i) *If $\theta \geq \frac{1}{2}(5+3\sqrt{2})c_2$, the subgame has an equilibrium. This equilibrium is not preferred by firm G to selling separate products.*

(ii) *If $\theta \leq \frac{3}{20}(2+\sqrt{2})c_2$, the subgame has an equilibrium. This equilibrium is preferred by firm G to selling separate products.*

(iii) *Otherwise the subgame has no equilibrium in pure strategies.*

Condition (i) corresponds to case (a3); condition (ii) corresponds to case (b3). In case (a3), there is no competition softening effect on the market for good X_2, and hence, firm G has no incentives to bundle. In case (b3), the effect is the same as discussed in the previous subsection for cases (b1) and (b2), i.e., the substitution effect is so strong that it causes a structural change allowing both firms A and G to relax their prices and earn higher profits. Similarly as in Proposition 4, the inequalities $\Pi_A > \Pi_G > \frac{1}{2}\theta$ hold in equilibrium, i.e., firm A is better off than firm G.

Much like in the previous subsection, the effect on both consumer surplus and total surplus is negative. When condition (ii) from Proposition 5 holds (the only case where pure bundling is preferred by firm G), the consumer surplus and total surplus are $CS = m+w-c_1+\frac{1}{2}(1-c_2)-\frac{179}{72}\theta$ and $TS = m+w-c_1+\frac{1}{2}(1-c_2)-\frac{31}{72}\theta$. Both are obviously lower than in the case of no bundling. By the same argument as in the previous subsection, this result is relevant for anti-trust policies (see also the discussion at the end of Subsection 3.2).

Remark 9. Concerning other types of mixed bundling, the intuition suggests that selling X_{G1} as well as the bundle may be preferred by firm G in cases (a1) or (a2) since it reduces the substitution effect. However, in cases (b1) and (b2), where the substitution effect is very strong, pure bundling should be preferred.

4 Continuous valuations of good X_2

In the basic model all consumers have the same valuation for good X_2 and prefer to buy it. In the extended model from the previous section, I considered two types of consumers — with valuation 0 and 1. In order to introduce more variety of decisions and to check the robustness of the previous section's conclusions, I will assume a continuous distribution of valuations for good X_2. I show that whenever the competition softening effect is significant, pure bundling is profitable when the substitution effect is weak (i.e., c_2 is small and θ is large), and that selling the bundle as well as X_2 is not profitable because of no competition softening effect. Moreover, I provide numerical examples to confirm the results of Propositions 4 and 5 and which illustrate that bundling has a negative effect on welfare whenever it is profitable for firm G.

In the model presented in this section, I assume that each consumer is indexed by a pair of parameters (α, v) which is uniformly distributed over the unit square; the parameters have the same meaning as in the basic model. Furthermore, I will assume that $0 < c_2 < 1$. The utility achieved by purchasing a particular combination of goods is the same as in Table 2. Similarly as in the previous section, I restrict the analysis to comparing the no bundling subgame with the pure bundling subgame and the mixed bundling subgame where firm G sells product X_2 as well as the bundle.[19]

4.1 No bundling

Analogically to Subsection 3.1, the consumers' decision which type of good X_1 to buy is the same as in the basic model, yielding the same equilibrium prices and profits as in (2). Bertrand competition on the market for good X_2 implies that $p_{G2} = p_{B2} = c_2$ in equilibrium, yielding zero profits. Hence, all consumers with $v \geq c_2$ buy good X_2. The consumer surplus in this case is $CS = m + w - c_1 + \frac{1}{2}(1-c_2)^2 - \frac{5}{4}\theta$; the total surplus is $TS = m + w - c_1 + \frac{1}{2}(1-c_2)^2 - \frac{1}{4}\theta$.

4.2 Pure bundling

When firm G engages in pure bundling, the situation changes due to the substitution effect. As compared to the the case of no bundling (independent pricing game), some consumers who had bought only good X_{G1} (i.e., they have valuation lower than the price of good X_2) also obtain good X_2 in the bundle. Therefore, they may prefer to switch to good X_{A1}. Clearly, consumer (α, v) buys the bundle if and only if $\alpha \leq \alpha^*(v)$, where $\alpha^*(v)$ is given by (3). All firms have a positive market share if and only if $0 < \alpha^*(p_{B2}) < 1$. Similarly as in the previous model,

[19] A similar discussion as in Remark 7 also applies here. Moreover, the other types of mixed bundling are not tractable in this model.

depending on the sign of $\alpha^*(0) = \frac{1}{2\theta}(\theta + p_{A1} - p_G)$, I obtain three cases for the distribution of the market:

$$\text{(a) } \alpha^*(0) > 0, \qquad \text{(b) } \alpha^*(0) < 0, \qquad \text{(c) } \alpha^*(0) = 0.$$

Figures 2 and 3 in Appendix B illustrate cases (a) and (b). The shaded area represents consumers who purchase the bundle. I will call an equilibrium *type* (a) *equilibrium*, if $\alpha^*(0) > 0$. Analogically, I define *type* (b) *equilibrium* and *type* (c) *equilibrium*. The market share[20] of firm G can be computed as follows:

$$q_G = \int_{\max\{0,\,-2\theta\alpha^*(0)\}}^{1} \alpha^*(v)\,\mathrm{d}v =$$

$$= \begin{cases} \frac{1}{2\theta}(\theta + p_{A1} - p_G + p_{B2} - \frac{1}{2}p_{B2}^2), & \text{if } \alpha^*(0) \geq 0, \\ \frac{1}{4\theta}(2 + \theta + p_{A1} - p_{B2} - p_G)(\theta + p_{A1} + p_{B2} - p_G), & \text{if } \alpha^*(0) \leq 0, \end{cases}$$

The market share of firm A is simply $q_{A1} = 1 - q_G$, and the market share of firm B is $q_{B2} = \big(1 - \alpha^*(p_{B2})\big)(1 - p_{B2}) = \frac{1}{2\theta}(-p_{A1} - p_{B2} + p_G + \theta)(1 - p_{B2})$. Therefore, the profits in case (a) are:

$$\Pi_G = \tfrac{1}{2\theta}(p_G - c_1 - c_2)\big(p_{A1} - p_G + \theta + p_{B2} - \tfrac{1}{2}p_{B2}^2\big), \tag{15}$$

$$\Pi_A = \tfrac{1}{2\theta}(p_{A1} - c_1)\big(-p_{A1} + p_G + \theta - p_{B2} + \tfrac{1}{2}p_{B2}^2\big), \tag{16}$$

$$\Pi_B = \tfrac{1}{2\theta}(p_{B2} - c_2)(-p_{A1} + p_G + \theta - p_{B2})(1 - p_{B2}). \tag{17}$$

Note that the formula for Π_B can be used only if $c_2 \leq p_{B2} \leq 1$ and $0 \leq \alpha^*(p_{B2}) \leq 1$. Obviously any p_{B2} outside the interval $[c_2, 1]$ is dominated. If $\alpha^*(p_{B2}) > 1$, the market share of firm B is zero and firm B is indifferent among all such prices. In such a case, I assume that it sets its price equal to c_2; see Remark 2.

Although the equilibrium prices of the pure bundling subgame cannot be specified explicitly as in previous models (see proof of the following lemma), I can prove some properties of the equilibrium. The profits in case (b) are much more complicated and no analogous statements can be obtained. Therefore, I will restrict further analysis to type (a) equilibria in which $\alpha^*(p_{B2}) < 1$.

Lemma 4. *In any type* (a) *equilibrium of the pure bundling subgame, firm B earns positive profit if and only if* $c_2^2 < 3\theta$. *In this case,* $c_2 < p_{B2} < \frac{1}{2}(1 + c_2)$.

Lemma 5. *There is no type* (a) *equilibrium of the pure bundling subgame where firm B earns zero profit.*

The above lemmas characterize type (a) equilibria of the pure bundling subgame.[21] The inequality $(1 - p_{B2})^2 \geq 1 + c_2 - 3\theta$ is a necessary condition for

[20] By market share, I understand the measure of all consumers who purchase the product.

[21] According to the above lemmas, $p_{B2} \leq \frac{1}{2}(1 + c_2)$ in equilibrium. This differs from the previous model, where no such restriction occurs and also equilibria with $p_{B2} = 1$ exist. The difference is caused by the form of the demand for good X_2. In this model it is continuous and equal to zero when $p_{B2} = 1$. However, in the previous section, I assumed the demand to be positive when $p_{B2} = 1$ causing a discontinuity (see also Remark 2).

such an equilibrium to exist. This was obtained by substitution of (20) and (21) from the proof of Lemma 4 into $\alpha^*(0) > 0$. Obviously, the inequality holds when $\theta > \frac{1}{3}(1 + c_2)$. Since it requires rather complicated computations to find an equivalent condition for type (a) equilibrium to occur in terms of parameters c_2 and θ, I performed numerical simulations whose results are sketched by the concave curve in Figure 4 in Appendix B. The region above the curve represents the values of parameters for which the necessary condition holds.

The following proposition shows a sufficient and necessary condition for profitability of type (a) equilibrium in the pure bundling subgame.

Proposition 6. *Firm G's profit in type* (a) *equilibrium of the pure bundling subgame is higher than in the equilibrium of the no bundling subgame if and only if* $c_2 < \bar{c}$ *and* $\theta > \bar{\theta}(c_2)$, *where* $\bar{c} = -3 + 2\sqrt{3} \approx 0.4641$ *and*

$$\bar{\theta}(c_2) = \frac{-c_2^2 + 8c_2 - 4 + 4(1 - c_2)\sqrt{1 - 2c_2}}{c_2 - 1 + 2\sqrt{1 - 2c_2}}$$

is an increasing function defined on the interval $[0, \bar{c}]$*; see Figure 4 in Appendix B.*

Remark 10. It can be easily shown that $\bar{\theta}(c_2) > \frac{1}{3}c_2^2$ for all $c_2 \in [0, \bar{c}]$. This means that any c_2 which satisfies the conditions from Proposition 6, also satisfies the necessary condition from Lemma 4.

Proposition 6 is clearly consistent with the result of Proposition 4 for type (a) equilibria. It implies that for fixed c_2 (such that $c_2 < \bar{c}$), firm G prefers type (a) equilibrium of the pure bundling subgame if the transportation costs θ are high enough to yield a small substitution effect. The intuition behind this result is the following. For small θ the products are less differentiated; therefore, consumers with low valuation for good X_2 who buy X_{G1} in the no bundling subgame switch easily to good X_{A1} in the pure bundling subgame. The function $\bar{\theta}$ determines the minimal value of θ such that firm G prefers pure bundling; Figure 4 in Appendix B shows its graph (the convex curve). Together with the necessary condition for the existence of type (a) equilibrium, the region above both curves represents the values of parameters where bundling is profitable in type (a) equilibrium. Example 1 below illustrates a particular case where pure bundling is preferred by firm G.

On the other hand, if I fix the value of θ, the above result can be formulated as the following. Firm G prefers pure bundling if the unit cost of good X_2 is low enough. The inverse of function $\bar{\theta}$ represents the maximal value of c_2 for which firm G prefers pure bundling. This maximal value is bounded from above by $\bar{c}$, which means that $\bar{c}$ is a critical value such that for any $c > \bar{c}$, and pure bundling is not profitable in type (a) equilibrium. The intuition behind this result is that for high values of c_2, the competition softening effect is weak and only allows a mark-up that is too small to make bundling profitable. Note also that the critical value of $\bar{c}$ is close to the critical value $\frac{1}{2}$ from previous model (Proposition 4).

Although I am not able to find the prices explicitly, I can evaluate them numerically for given values of c_2 and θ. The following two examples illustrate two types of equilibria. In the first one, I consider a high value of θ and a low value of c_2 to illustrate the type (a) equilibrium and the results of Lemma 4 and Proposition 6. In the second example, I consider a low value of θ yielding a type (b) equilibrium. In this case, all firms are better off in equilibrium, which is consistent with Proposition 4.

Example 1. Consider the values $c_2 = 0.4$ and $\theta = 1$ (and $c_1 = 0$ for simplicity). In the no bundling subgame, profits of firms A and G are $\frac{1}{2}\theta = \frac{1}{2}$. To find out whether pure bundling is profitable, I apply Proposition 6 and obtain $\bar{\theta}(c_2) = 0.3849$. Therefore, pure bundling should be more profitable for firm G than selling separate products. The equilibrium prices and profits can be computed numerically from (20), (21), and (24); see Table 4 in Appendix B. Hence, pure bundling is indeed preferred by firm G to selling separate products. Compared to the consumer surplus $m + w - 1.070$ and the total surplus $m + w - 0.070$ in the no bundling subgame, both are lower in the case of pure bundling, which confirms the results of the previous model with two types of consumers.

Example 2. Consider the values $\theta = 0.01$ and $c_2 = 0.4$ (and again $c_1 = 0$ for simplicity). In the no bundling subgame, profits of firms A and G are $\frac{1}{2}\theta = 0.005$. Table 4 in Appendix B shows the equilibrium prices and profits in the pure bundling subgame. Obviously all firms earn higher profit than in the case of no bundling. Compared to the consumer surplus $m+w+0.1675$ and the total surplus $m + w + 0.1775$ in the no bundling subgame, both are lower in the case of pure bundling, which confirms the results of the previous model.

4.3 Mixed bundling by firm G

When firm G decides to sell the bundle as well as good X_2, by an analogical argument (Bertrand competition) as in the basic model, I obtain that $p_{G2} = p_{B2} = c_2$ in equilibrium. Firms G and A maximize the same profit function as specified in (15) and (16), with $p_{B2} = c_2$. This is an analogous situation to the one analyzed in Lemma 5. In its proof, I have shown that

$$c_2(3 - c_2) \leq 3\theta$$

is a necessary condition for $\alpha^*(0) > 0$. Hence, if it is violated, there is no type (a) equilibrium. Otherwise, I can evaluate the equilibrium prices and profits for type (a) equilibrium as specified in the following proposition.

Proposition 7. *In a type* (a) *equilibrium of the subgame when firm G offers the bundle and good X_2, the equilibrium prices are*

$$p_G = \theta + c_1 + c_2 - \frac{c_2^2}{6}, \qquad p_{A1} = \theta + c_1 + \frac{c_2^2}{6}, \qquad p_{G2} = p_{B2} = c_2, \tag{18}$$

yielding firm G's market share $\frac{1}{2} - \frac{1}{6\theta}c_2^2$ and profits

$$\Pi_G = \frac{1}{2\theta}\left(\theta - \frac{c_2^2}{6}\right)^2, \qquad \Pi_A = \frac{1}{2\theta}\left(\theta + \frac{c_2^2}{6}\right)^2, \qquad \Pi_B = 0. \tag{19}$$

The proposition shows that $\Pi_A > \frac{1}{2}\theta > \Pi_G$, which confirms the intuition that this type of bundling is not profitable because of the absence of the competition softening effect. On the other hand, the substitution effect is present (i.e., there are consumers who prefer X_{G1} over X_{A1}, but do not want good X_2, so they switch to X_{A1}, and it does not pay for firm G to lower the price sufficiently to compensate them). However, when c_2 approaches zero, the measure of the consumers who do not want good X_2 approaches zero and the substitution effect disappears, making firm G indifferent between this type of bundling and selling separate products. An analogous result can be obtained by considering valuations for good X_2 from some interval $[\underline{v}, 1]$, where $\underline{v} \geq c_2$.

Remark 11. The result can be again interpreted as the equilibrium when the market for good X_2 is perfectly competitive (see Corollary 1). In this case, there is obviously no competition softening effect in type (a) equilibrium. Hence, pure bundling is unprofitable.

Similarly as in the previous subsection, it is not possible to evaluate the type (b) equilibrium prices in terms of parameters. However, I provide a numerical example to support the result of Proposition 5, which shows that selling good X_2 and the bundle may be profitable for firm G when θ is low.

Example 3. Consider again the same values as in Example 2: $\theta = 0.01$ and $c_2 = 0.4$ (and $c_1 = 0$). In the no bundling subgame, profits of firms A and G are $\frac{1}{2}\theta = 0.005$. Table 4 in Appendix B describes the equilibrium prices and profits in this subgame. Obviously, all firms earn higher profit than in the case of no bundling. Compared to the consumer surplus $m + w + 0.1675$ and the total surplus $m + w + 0.1775$ in the no bundling subgame, both are lower in the case of mixed bundling, which confirms the results of the previous model.

5 Conclusion

In this paper, I analyze bundling by a multi-product (generalist) firm competing against several single-product (specialist) firms. I consider the case of two markets: a duopoly for a heterogeneous good, and a duopoly or perfect competition for a homogeneous good. When the generalist firm decides to bundle, two effects emerge: the competition softening effect and the substitution effect.

I show that in the case of duopoly on the market for the second good, if consumers' valuations for it are homogeneous and high enough, there is no substitution effect and the generalist firm chooses pure bundling in equilibrium. However, this strategy has a negative effect on the rival and a negative welfare effect. This result is important for anti-trust policies because it shows that a firm facing an equal rival may also abuse its position as a multi-product firm. This finding is at variance with the widely accepted argument by Posner (1976) that tying may be profitable only in the case of monopoly, and it should be taken into account by anti-trust authorities.

Moreover, if pure bundling is banned, firm G may also achieve the same outcome when it offers the heterogeneous good for a high price such that nobody wants to buy it separately. However, this price can be even lower than the equilibrium price when selling separate products. This suggests an interesting policy implication that it may not be sufficient to ban only pure bundling since the generalist firm may replicate the same outcome via mixed bundling.

In order to illustrate the substitution effect, I extend the model by considering two types of consumers: with low and high valuation for the second good (later, I check the robustness of these results by considering continuous valuations). In particular, I show that pure bundling is preferred by the generalist firm to selling the products separately in the case where the competition softening effect is significant and the substitution effect is weak and in the case where the substitution effect is very strong because it changes the structure of the market by segmenting the market. In the latter case, pure bundling gives a rival producing the first good certain market power over consumers with a low valuation for the second good. Hence, the prices can rise resulting in higher profits for all three firms. This is the most striking result of this paper. It suggests that in certain cases all firms can be better off, but at the expense of consumer surplus and social welfare. Hence, no firm is willing to start a case against a multi-product firm. Anti-trust authorities should therefore play a more active role in identifying cases where bundling harms consumers (and society) and cannot rely on rivals' appeals.

In the case of perfect competition on the market for the second good, there is no competition softening effect, and the generalist firm is indifferent between pure bundling and selling separate products when the valuations for the second good are homogeneous. However, when they are heterogeneous, selling separate products is preferred by the generalist firm unless the substitution effect is so strong that it changes the structure of the market.

This paper makes an important step in exploring the abuse of tying by a multi-product firm without monopoly power. Its results are relevant for anti-trust policies since they indicate that a firm which faces an equal competitor in each market can successfully use tying, which consequently causes welfare losses. Hence, tying should not be considered as abuse of monopoly (or of a dominant) position, but more as an abuse of a firm's position as a multi-product firm. In the future, the understanding of this issue should be extended in several directions:

- First, the fact that the multi-product firm can increase its profit by tying raises the obvious question whether it can even force foreclosure of some of its rivals.

- Second, in the paper I consider two markets with particular structures. A higher number of markets with different structures should be analyzed to see how robust the results of this paper are.

- Third, in the extension of the basic model, I analyze only one type of mixed bundling because of its important interpretation. It would be indeed useful to completely analyze the equilibria in the mixed bundling subgame and the subgame-perfect equilibrium.

- Fourth, a relevant question is how do the results change when unbundling and resale on a secondary market are possible.

Such deep analysis would improve our understanding of bundling, shedding light on many still controversial anti-trust cases.

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A Appendix: Proofs

Proof of Lemma 1. As already mentioned, the consumer chooses only between buying both goods X_{A1}, X_2 or the bundle $\mathcal{G} = \{X_{G1}, X_2\}$ yielding the market share α^* for firm G. Maximization of profits (4), (5), and (6) with respect to appropriate prices (note that all profit functions are concave) leads to first order conditions

$$\begin{aligned} p_{A1} + p_{B2} - 2p_G + \theta + c_1 + c_2 &= 0, \\ -2p_{A1} - p_{B2} + p_G + \theta + c_1 &= 0, \\ -p_{A1} - 2p_{B2} + p_G + \theta + c_2 &= 0, \end{aligned}$$

which are linear equations of unknowns p_{A1}, p_{B2}, p_G. By solving them I obtain the equilibrium prices (7). Note that the prices are always higher than unit costs. Obviously, $p_{B2} \leq v$ in equilibrium, meaning that this p_{B2} is indeed the maximum of Π_{B2} on the interval $[0, v]$.

To prove that the prices (7) establish a Nash equilibrium, I will show that no firm has incentives to deviate by undercutting (i.e., to yield another firm's profit zero). Consider p_{A1} and p_{B2} given by (7). When firm G decides to undercut, it has the highest profit if $\alpha^* = 1$, i.e., $p_G = p_{A1} + p_{B2} - \theta = \frac{1}{2}\theta + c_1 + c_2$. However, because the firms are located at the edges of the unit interval, the profit function is continuous[22] so the profit is lower than the interior maximum attained for $p_G = \frac{5}{4}\theta + c_1 + c_2$. This can also be verified by a direct computation. Similarly, if firm A or B wants to undercut, then undercutting yields the highest profit when $\alpha^* = 0$. By an analogical argument as above, undercutting is not profitable for any of them.

The above equilibrium was derived under the assumption that $0 < \alpha^* < 1$ (i.e., all firms have a positive market share). To complete the proof, I will show that there is no other equilibrium. If $\alpha^* \geq 1$, in equilibrium, then $\alpha^* = 1$. Otherwise firm G, which captures the whole market, can increase its price to achieve a higher profit. For $\alpha^* = 1$, i.e., $p_G - p_{A1} - p_{B2} + \theta = 0$, it may not be profitable for firm A to decrease its price. Hence, $\partial\Pi_A/\partial p_{A1}|_{\alpha^*=1} \geq 0$ or $p_{A1} = c_1$. However, the first condition yields $p_{A1} \leq c_1$, which means that $p_{A1} = c_1$. Analogically, I obtain $p_{B2} = c_2$, and hence, $p_G = c_1 + c_2 - \theta < c_1 + c_2$. Therefore, there is no equilibrium such that $\alpha^* \geq 1$. By a similar argument, I can show that there is no equilibrium such that $\alpha^* \leq 0$. □

Proof of Lemma 2. Consider an equilibrium in which $p_{B2} = v$. The profit functions and also the first order conditions for firms G and A are the same as in the previous proof. Solving the system, I obtain the prices given by (8); an easy check shows that $0 < \alpha^* < 1$. To show that $p_{B2} = v$ is also the best response to firm A's and G's prices (8), I substitute them into B's profit function to obtain $\partial\Pi_B/\partial p_{B2} = \frac{1}{2\theta}(-2p_{B2} + \frac{2}{3}v + \frac{4}{3}c_2 + \theta)$. This means that Π_B is increasing on the interval $[c_2, v]$ and henceforth attains its maximum for $p_{B2} = v$. Similarly as in the proof of the previous lemma, I can argue that no firm has incentives to undercut and that there is no equilibrium such that $\alpha^* \leq 0$ or $\alpha^* \geq 1$. □

[22] In the Hotelling model, the profit function may be discontinuous in the point of undercutting when the firm is located inside the interval. See, for example, Shy (1996), p. 163.

Proof of Proposition 1. Obviously, the prices are obtained directly from (7) and (8). These yield the market share $\alpha^* = \frac{1}{2} + \frac{1}{2\theta}\mu$, and after substitution into the profit functions from (5), (6), and (4), they yield the profits (11). Finally, the consumer surplus can be evaluated as

$$\begin{aligned} CS &= \int_0^{\frac{1}{2}+\frac{1}{2\theta}\mu} \big(m + w - (\theta + c_1 + c_2 + \mu) - \theta\alpha + v\big)\, \mathrm{d}\alpha + \\ &+ \int_{\frac{1}{2}+\frac{1}{2\theta}\mu}^{1} \big(m + w - (\theta + c_1 - \mu) - \theta(1-\alpha) - (c_2 + 3\mu) + v\big)\, \mathrm{d}\alpha = \\ &= m + w + v - c_1 - c_2 - \tfrac{5}{4}\theta - \tfrac{3}{2}\mu + \tfrac{1}{4\theta}\mu^2, \end{aligned}$$

which completes the proof. □

Proof of Proposition 2. Obviously, there is no equilibrium where $p_{B2} = c_2$. Hence, $p_{B2} > c_2$ in equilibrium. If $p_G > p_{G1} + p_{B2}$, no one buys the bundle and everyone buys good X_2 from firm B. In this case firm G may decrease p_G below $p_{G1} + p_{B2}$, and all consumers buying X_{G1} and X_{B2} switch to the bundle. The same argument also applies to the case where $p_G = p_{G1} + p_{B2}$ (see Remark 1). Moreover in this case, firm B may slightly decrease its price, and all consumers buying the bundle switch to goods X_{G1} and X_{B2}. Therefore, $p_G < p_{G1} + p_{B2}$ in equilibrium, which means that no one wants to buy good X_{G1} separately (not in the bundle). This yields the same equilibrium prices p_G, p_{A1}, p_{B2} as in the pure bundling subgame.

Now it remains to derive the condition for p_{G1}. The above analysis implies that $p_{G1} > p_G - p_{B2} = \theta + c_1 - 2\mu$. Despite this, firm B may be willing to decrease its price to $p'_{B2} < p_G - p_{G1}$ in which case all consumers buy good X_{B2} yielding firm B's profit $\Pi'_B = p'_{B2} - c_2$. Firm B does not prefer to do so if and only if $p_G - p_{G1} - c_2 < \Pi_B$. Substituting the prices and profits from Proposition 1, I obtain condition (12). One can easily check that $p_G - p_{B2} = \theta + c_1 - 2\mu$ is lower than the right-hand side of (12), which means that the condition implies $p_G < p_{G1} + p_{B2}$. □

Proof of Proposition 3. If $\alpha < 1$ and $p_{G2}, p_{B2} > c_2$, the firm with the higher price may undercut its opponent to capture the whole market for good X_2 of measure $1 - \alpha^*$. On the other hand, if $p_{B2} > p_{G2} = c_2$, then firm G earns zero profit from selling good X_2, but it may increase p_{G2} which increases α^* and its profits (both from selling the bundle and good X_2). If $p_{G2} > p_{B2} = c_2$, then firm B will increase its profit by increasing p_{B2}. This proves that $p_{G2} = p_{B2} = c_2$ in equilibrium.

Obviously, the marginal consumer is determined by

$$\alpha^* = \frac{p_{A1} - (p_G - c_1)}{2\theta} + \frac{1}{2}.$$

Hence, the profits of firms G and A are the same as in the no bundling subgame when $p_{G1} = p_G - c_2$. This yields the equilibrium prices (13).

Similarly as in the proof of Lemma 1, I can argue that both G and A have no incentives to capture the whole market for good X_1, which would cause $\alpha^* \notin (0, 1)$. □

Proof of Lemma 3. In any type (c) equilibrium, $\partial\Pi_G/\partial p_G|_{p_G=p_{A1}+\theta}$ must be non-negative from left and non-positive from right. Taking $p_G \to p_{A1}+\theta$ in (14) gives $p_{B2} \le 0$, which is a contradiction. Hence, there is no type (c) equilibrium. □

Proof of Proposition 4. The proof directly follows from Table 3 in Appendix B. □

Proof of Proposition 5. The proof directly follows from the discussion preceding the proposition and Table 3 in Appendix B. □

Proof of Lemma 4. The first order conditions for the maximization of profits of firms G and A are

$$\begin{aligned} p_{A1} + p_{B2} - 2p_G + \theta + c_1 + c_2 - \tfrac{1}{2}p_{B2}^2 &= 0, \\ -2p_{A1} - p_{B2} + p_G + \theta + c_1 + \tfrac{1}{2}p_{B2}^2 &= 0, \end{aligned}$$

which yield

$$\begin{aligned} p_G &= \theta + c_1 + \tfrac{2}{3}c_2 + \tfrac{1}{3}p_{B2} - \tfrac{1}{6}p_{B2}^2, \quad (20)\\ p_{A1} &= \theta + c_1 + \tfrac{1}{3}c_2 - \tfrac{1}{3}p_{B2} + \tfrac{1}{6}p_{B2}^2. \quad (21) \end{aligned}$$

Firm B's profit function (17) has one of the shapes sketched in Figure 5 in Appendix B, depending on the position of its zero points c_2, 1, and $p_G - p_{A1} + \theta$. It has to be maximized on the interval $[c_2, 1]$. Consider first an interior solution. Differentiating with respect to p_{B2}, I obtain

$$\begin{aligned} \frac{\partial \Pi_B}{\partial p_{B2}} &= \frac{1}{2\theta}[(1-p_{B2})(p_G+\theta-p_{A1}-p_{B2}) - (p_{B2}-c_2)(1-p_{B2}) - \\ &\quad -(p_{B2}-c_2)(p_G+\theta-p_{A1}-p_{B2})], \end{aligned} \quad (22)$$

which is quadratic in p_{B2}. Obviously $\partial\Pi_B/\partial p_{B2}|_{p_{B2}=\frac{1}{2}(1+c)} = -\frac{1}{8\theta}(1-c_2)^2 < 0$. Hence (see also Figure 5 in Appendix B), it is clear that a solution of (22) is a local maximum of Π_B function if and only if

$$p_{B2} < \frac{1+c_2}{2}. \quad (23)$$

I prefer this condition because it is simpler than the second order condition.

After substituting the solutions (20) and (21) for p_G and p_{A1} into the first order condition (22), I obtain that p_{B2} is a solution of a cubic equation $f(x) = 0$ where

$$f(x) = 2x^3 + (4-c_2)x^2 - 2(3c_2 + 3\theta + 2)x + (c_2^2 + 4c_2 + 3\theta + 3c_2\theta). \quad (24)$$

This solution establishes an equilibrium of the pure bundling subgame if and only if it belongs to the interval $J = \big(c_2,\ \frac{1}{2}(1+c_2)\big)$. Cubic equations can indeed be solved, but the formula is too complicated for any further analysis. Therefore, I do not compute the exact solution, but I provide a sufficient and necessary condition for its root to belong to J. Obviously, $\lim_{x\to\pm\infty} f(x) = \pm\infty$, $f(0) > 0$, and $f\big(\frac{1}{2}(1+c_2)\big) < 0$. This means that the equation $f(x) = 0$ has three real solutions: the first of them in $(-\infty, 0)$; the second in $\big(0,\ \frac{1}{2}(1+c_2)\big)$; and the third in $\big(\frac{1}{2}(1+c_2),\ +\infty\big)$. The second root lies in J (the others obviously lie outside J) if and only if $f(c_2) > 0$, which is equivalent to

$$c_2^2 < 3\theta. \quad (25)$$

This proves the lemma. □

Proof of Lemma 5. If firm B earns zero profit in equilibrium, then $p_{B2} = c_2$ (see Remark 2). If $p_{B2} = c_2$ in a type (a) equilibrium, then p_G and p_{A1} are given by G's and A's best responses (20) and (21). Hence, $p_G = \theta + c_1 + c_2 - \frac{c_2^2}{6}$, and $p_{A1} = \theta + c_1 + \frac{c_2^2}{6}$. To find firm B's best response to those prices, I substitute them into B's profit function to obtain $\Pi_B = (p_{B2} - c_2)(c_2 - \frac{1}{3}c_2^2 + \theta - p_{B2})(1 - p_{B2})$. This is negative for all $p_{B2} \in (c_2, 1)$ if and only if $c_2 - \frac{1}{3}c_2^2 + \theta \leq c_2$, i.e., $3\theta \leq c_2^2$.

For the above prices $\alpha^*(0) = \frac{1}{2\theta}(\theta - c_2 + \frac{1}{3}c_2^2)$, which is non-negative if and only if $3c_2 - c_2^2 \leq 3\theta$. However, this condition cannot hold together with $3\theta \leq c_2^2$ because $3c_2 - c_2^2 > c_2^2$ for any $c_2 \in (0, 1]$. Hence, there is no type (a) equilibrium such that $p_{B2} = c_2$. □

Proof of Proposition 6. Because the prices cannot be computed explicitly (see the proof of Lemma 4), I will use p_{B2} as a parameter. This way, I identify all values of p_{B2} for which the type (a) equilibrium profit in the pure bundling subgame is higher than in the no bundling subgame. After substituting the G's and A's best responses (20) and (21) into G's profit function, I obtain firm G's equilibrium profit expressed in terms of p_{B2}:

$$\Pi_G = \frac{1}{2\theta}\left(\theta + \frac{p_{B2} - c_2}{3} - \frac{p_{B2}^2}{6}\right)^2.$$

This is higher than $\frac{1}{2}\theta$ (profit in the case of no bundling) if and only if $2(p_{B2} - c_2) > p_{B2}^2$, or $1 - 2c_2 > (1 - p_{B2})^2$ which never holds for $c_2 \geq \frac{1}{2}$. On the other hand, if $c < \frac{1}{2}$, the above inequality is equivalent to $1 - \sqrt{1 - 2c_2} < p_{B2} < 1 + \sqrt{1 - c_2}$. Obviously, the lower bound is higher than c_2, and the upper bound is higher than 1. Therefore, for $p_{B2} \in [c_2, 1]$, pure bundling is more profitable for firm G if and only if

$$p_{B2} \geq 1 - \sqrt{1 - 2c_2}. \tag{26}$$

Note that $c_2 \leq 1 - \sqrt{1 - 2c_2} \leq 1$. This, together with the fact that function f defined by (24) from the proof of Lemma 4 is positive on (c_2, p_{B2}) and negative on $\left(p_{B2}, \frac{1}{2}(1 + c_2)\right)$, implies that (26) holds if and only if the following conditions are satisfied:

$$1 - \sqrt{1 - 2c_2} \quad < \quad \frac{1 + c_2}{2}, \tag{27}$$

$$f\left(1 - \sqrt{1 - 2c_2}\right) \quad > \quad 0. \tag{28}$$

Condition (27) is equivalent to $c_2 < -3 + 2\sqrt{3} = \bar{c}$ and condition (28) is equivalent to $\theta > \bar{\theta}(c_2)$ whenever $c_2 < -3 + 2\sqrt{3}$.

To complete the proof, I will show that $\bar{\theta}$ is increasing on $[0, \bar{c}]$. Straightforward computations yield that $\mathrm{d}\bar{\theta}(c_2)/\mathrm{d}c_2 \to \infty$ as $c \to \bar{c}$ from left and $\mathrm{d}\bar{\theta}(c_2)/\mathrm{d}c_2|_{c_2=0} = 0$. Moreover, it can be easily shown that $\mathrm{d}\bar{\theta}(c_2)/\mathrm{d}c_2$ is defined and continuous on $[0, \bar{c}]$ and has no root in $(0, \bar{c}]$. □

Proof of Proposition 7. The equality $p_{B2} = c_2$ follows from the discussion preceding the proposition. The prices are evaluated in the proof of Lemma 5. All remaining computations are straightforward. □

B Appendix: Figures and Tables

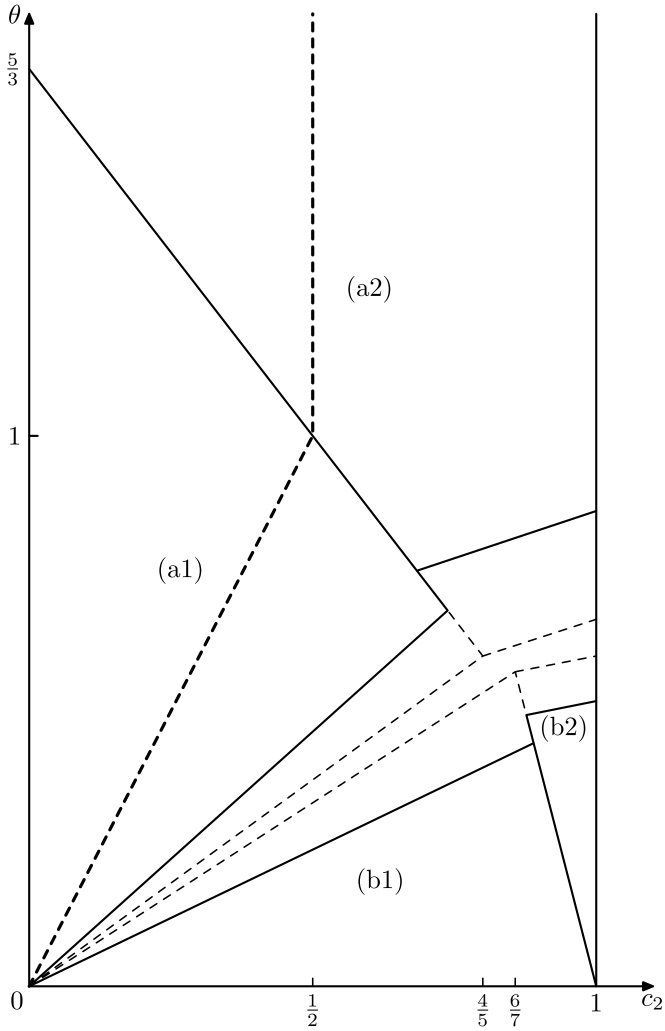

Figure 1: Equilibria in the case of heterogeneous valuations for good X_{G2}

Equilibrium type		$p-MC$ for A,B,G	Necessary conditions	Profits Π_A, Π_B, Π_G	No switch condition, bundling decision
(a1)	$\alpha^*(0)>0$	$\frac{1}{10}(9\theta+2c_2)$	$\theta>\frac{3}{4}c_2$	$\frac{1}{200\theta}\ (9\theta+2c_2)^2$	$\theta\geq\frac{4}{343}(44+25\sqrt{2})c_2$
	$c_2<p_{B2}<1$	$\frac{1}{5}(3\theta-c_2)$	$\theta<\frac{1}{3}(5-4c_2)$	$\frac{1}{200\theta}\ (3\theta-c_2)^2$	
		$\frac{1}{10}(11\theta-2c_2)$	needs $c_2<\frac{4}{5}$	$\frac{1}{200\theta}\ (11\theta-2c_2)^2$	bundle iff $\theta\geq 2c_2$
(a2)	$\alpha^*(0)>0$	$\theta-\frac{1}{6}(1-2c_2)$	$\theta>\frac{1}{3}(1+c_2)$ for $c_2\geq\frac{4}{5}$	$\frac{1}{72\theta}\ (6\theta-1+2c_2)^2$	$\theta\geq\frac{1}{3}c_2+\frac{1}{12}(2+3\sqrt{2})$
	$p_{B2}=1$	$1-c_2$	$\theta\geq\frac{1}{3}(5-4c_2)$ for $c_2<\frac{4}{5}$	$\frac{1}{12\theta}\ (1-c_2)(3\theta-2+c_2)$	
		$\theta+\frac{1}{6}(1-2c_2)$		$\frac{1}{72\theta}\ (6\theta+1-2c_2)^2$	bundle iff $c_2\leq\frac{1}{2}$
(a3)	$\alpha^*(0)>0$	$\theta+\frac{1}{6}c_2$	$\theta>\frac{2}{3}c_2$	$\frac{1}{72\theta}\ (6\theta+c_2)^2$	$\theta\geq\frac{1}{2}(5+3\sqrt{2})c_2$
	$p_{B2}=c_2$	0	never holds	0	
		$\theta-\frac{1}{6}c_2$		$\frac{1}{72\theta}\ (6\theta-c_2)^2$	never bundle
(b1)	$\alpha^*(0)<0$	$\frac{9}{4}\theta$	$\theta<\frac{2}{3}c_2$ for $c_2<\frac{6}{7}$	$\frac{81}{64}\ \theta$	$\theta\leq\frac{4}{337}(27+14\sqrt{2})c_2$
	$c_2<p_{B2}<1$	$\frac{1}{4}\theta$	$\theta<4(1-c_2)$ for $c_2\geq\frac{6}{7}$	$\frac{1}{64}\ \theta$	
		$\frac{7}{4}\theta$		$\frac{49}{64}\ \theta$	always bundle
(b2)	$\alpha^*(0)<0$	$\frac{1}{3}(7\theta+c_2-1)$	$\theta<\frac{1}{5}(2+c_2)$	$\frac{1}{36\theta}\ (7\theta-1+c_2)^2$	$\theta\leq\frac{1}{5}c_2+\frac{1}{20}(2+3\sqrt{2})$
	$p_{B2}=1$	$1-c_2$	$\theta\geq 4(1-c_2)$	$\frac{1}{12\theta}\ (1-c_2)(\theta+1-c_2)$	
		$\frac{1}{3}(5\theta+1-c_2)$	needs $c_2>\frac{6}{7}$	$\frac{1}{36\theta}\ (5\theta+1-c_2)^2$	always bundle
(b3)	$\alpha^*(0)<0$	$\frac{7}{3}\theta$	$\theta<\frac{3}{5}c_2$	$\frac{49}{36}\ \theta$	$\theta\leq\frac{3}{20}(2+\sqrt{2})c_2$
	$p_{B2}=c_2$	0	never holds	0	
		$\frac{5}{3}\theta$		$\frac{25}{36}\ \theta$	always bundle

Table 3: Equilibria in the case of heterogeneous valuations for good X_{G2}

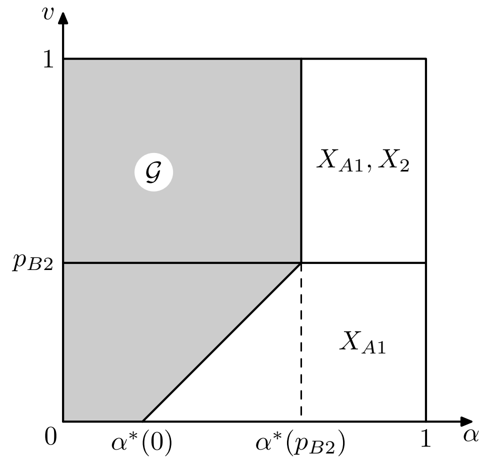

Figure 2: Distribution of the market: Case (a) $\alpha^*(0) > 0$

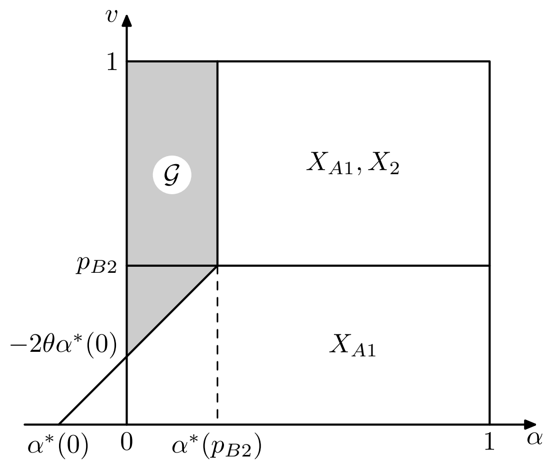

Figure 3: Distribution of the market: Case (b) $\alpha^*(0) < 0$

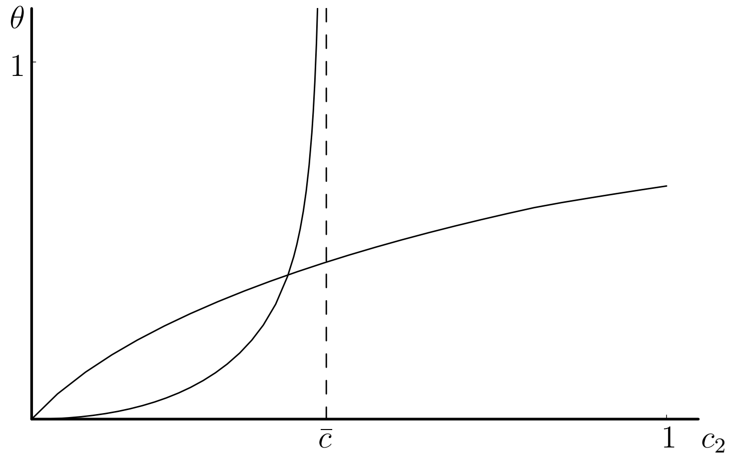

Figure 4: Graph of the function $\hat{\theta}$

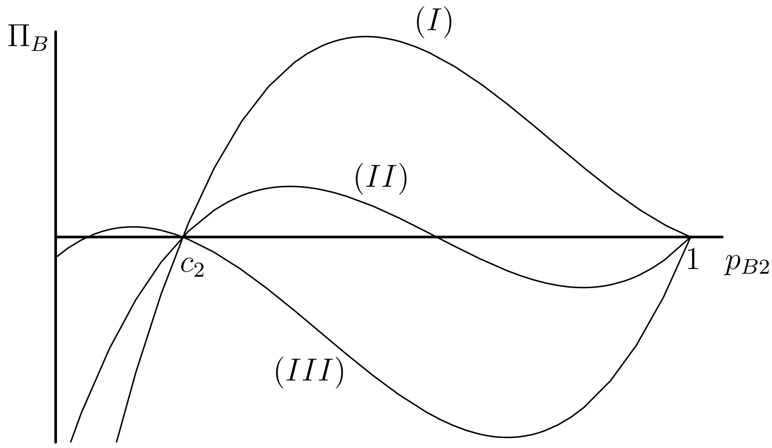

Figure 5: Possible shapes of Π_B

Ex.	Case, parameters	Prices A, B, G	$\alpha^*(0)$, $\alpha^*(p_{B2})$	Profits A, B, G	CS, TS
1	pure bundling	0.9878	0.2878	0.4878	$m + w - 1.1568$
	type (a) equil.	0.6443	0.6099	0.0339	$m + w - 0.1228$
	$c_2 = 0.4$, $\theta = 1$	1.4122		0.5123	
2	pure bundling	0.0171	-19.4215	0.0090	$m + w + 0.1327$
	type (b) equil.	0.4042	0.7885	0.0005	$m + w + 0.1596$
	$c_2 = 0.4$, $\theta = 0.01$	0.4156		0.0074	
3	mixed bundling	0.0184	-19.2852	0.0104	$m + w + 0.1210$
	type (b) equil.	0.4	0.7148	0	$m + w + 0.1375$
	$c_2 = 0.4$, $\theta = 0.01$	0.4141		0.0061	

Table 4: Numerical examples