Financing of Competing Projects with Venture Capital*

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Abstract

We analyze innovation races in a moral hazard setting. We develop a model where two competing entrepreneurs work independently on the same project. The entrepreneurs do not possess any wealth of their own and their research is financed by a venture capitalist. The project, if successful, generates a prize, which is to be shared between the winning entrepreneur and the venture capitalist. The venture capitalist cannot observe the allocation of funds he provides, which creates a moral hazard problem. We compare a competitive setting with a benchmark case where the venture capitalist finances only one entrepreneur. We show that the venture capitalist can increase the efficiency of research (hence, his own expected profit from investments) and alleviate the moral hazard problem, if he finances both entrepreneurs. This conclusion is unambiguous, when the entrepreneurs are at the same stage of R&D. Moreover, it holds for a large range of parameters also when the entrepreneurs are at the different stages of R&D, so that one of them is the leader in the innovation race and the other is the follower.

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1 Introduction

The classical approach in the literature on patent races is to model firms run by their owner and to assume away any incentive problems within these firms.[1] This approach ignores an important fact that in many cases the financing and allocation decisions in R&D process are separated. This problem arises within firms (for example, if they subcontract R&D) but it is especially important for grant agencies and venture capital funds.

Venture capital funds are usually directed to projects of uncertain quality, where neither time nor financial recourses needed for successful completion of the project are known *ex ante*. As a rule, venture capitalists are actively involved in monitoring firms in their portfolio. Nevertheless, they can rarely control perfectly whether resources are allocated efficiently, since such control requires an expertise which often only an entrepreneur himself possesses. This creates a moral hazard problem: entrepreneurs tend to misallocate the funds provided by the venture capitalist. In particular, they may divert part of funds for their own uses, or may allocate them into activities, which have high personal return but create little market value (Gompers and Lerner 2004, p. 174).

The venture capital literature has extensively discussed contractual arrangements that can be used by in order to alleviate the moral hazard problem. These are, for example, convertible securities (Sahlman 1990, Kaplan and Stromberg 2003) and monitoring mechanisms (Gompers 1995). On one hand, these mechanisms are efficient in mitigating the agency conflict. On the other hand, they are costly, complicated, and in some circumstances they are not feasible at all (most obviously, if the capital markets are not sufficiently developed to allow the use of complicated securities schemes). This creates obstacles for efficient funding of R&D.

This paper, as opposed to the existing literature on venture capital, proposes a non-contractual mechanism, namely competition between portfolio firms, that can be used to mitigate the agency conflict. The main question that we address is whether a venture capitalist can use competition between portfolio entrepreneurs to improve their incentives and thus his own profit. To answer this question we investigate a patent race in a moral hazard setting, where financing decision (made by the venture capitalist) and the allocation decisions (made by two competing agents) are separated.

Comparing the patent race with a basic set up where only one agent is employed, we identify two effects of competition. Obviously, competition allows to increase the probability of success, since two agents succeed (on average) more often than one (*scale effect*). But, more importantly, the fear that the competitor wins the patent race limits the option of the agents to divert funds for own uses (*disciplining effect*). While the scale effect is important when the agents are symmetric, we show that it plays negligible role when the agents are asymmetric, so that one of them is the leader in the innovation race and the other is the follower. Nevertheless, the venture capitalist will often employ the follower together with the leader, although in terms of success probability the contribution of the former is minor. His presence, however, disciplines the leader and limits the rent

[1] Loury 1979, Lee and Wilde 1980, Reinganum 1981, Grossman and Shapiro 1987, Choi 1991, Malueg and Tsutsui 1997, etc.

which the latter can extract from the principal.

The observation that venture capital firms, grant agencies and similar institutions may find it profitable to finance an innovation race is well supported by casual empirical evidence. The venture capital division of Vulcan Inc., a corporation owned by Microsoft co-founder Paul Allen, has contracted three competing agencies for the project Halo, aimed at the development of the problem-solving software.[2] National Archives and Records Administration of the USA awarded two contracts to the competing firms to develop an Electronic Record Archives, a revolutionary system of record keeping.[3] National Institutes of Health (USA) routinely finances competing research teams working on the same problem.[4]

Despite the numerous evidence that financing of competing projects plays an important role in the venture capital and the grant funding, the literature on venture capital has up to now paid little attention to this potent mechanism of mitigating the agency problem. The only exception is Levitt (1995), who analyzes the problem of a principal employing two agents. The principal's payoff depends on the best of agents' outputs. Unlike in our model, the author investigates a static situation that does not allow to address a competition between the leader and the follower. Moreover his results rely on the fact that production technologies of the agents are interdependent, and the model is not able to deliver predictions for the case of independent technologies. We address both issues in our paper.

Analyzing the innovation race between two competing entrepreneurs we consider a research process consisting of several sequential stages that are observable and verifiable outcomes of R&D, such as a patent, results of tests, etc. Both entrepreneurs are financed by a single venture capitalist, who incurs the research costs and rewards the entrepreneurs if they succeed. He also determines the time horizon during which the project will be financed. Within this structure we investigate the effect of competition in two scenarios: when the entrepreneurs are at the same stage or at different stages of research. The simplest situation that allows to analyze the first scenario is the innovation race between two identical entrepreneurs who need to finish one stage of R&D in order to complete a project. In this case, the scale effect is important and in the moral hazard setting is re-enforced by the disciplining effect. We conclude that with identical entrepreneurs, competition is unambiguously beneficial for the venture capitalist: he will always prefers to employ two entrepreneurs rather then one.

The simplest situation which allows to analyze the second scenario is the research process consisting of two sequential stages. In particular, we assume that the venture capitalist has in his portfolio a *leader*, who has successfully completed the first stage of R&D. The venture capitalist now faces an opportunity to finance a second entrepreneur, a *follower*, who is in the initial stage of R&D. Should the venture capitalist employ both agents, or should he proceed financing the leader alone? It turns out, that the scale effect

[2]See `www.projecthalo.com` for details

[3]For details see `www.diglib.org/preserve/ERA2004.htm`.

[4]Recent example involves two large scale competing studies which independently revealed a gene responsible for multiple sclerosis. Both studies were supported by a grant from NIH. For details see `www.ninds.nih.gov/ news and events/press releases/`.

is negligible. Therefore, in the absence of the moral hazard, the venture capitalist almost always prefers to employ the most advanced agent, the leader. In the presence of the moral hazard, however, the existence of the follower is important because it relaxes the incentive compatibility constraint of the leader. Due to this disciplining effect the venture capitalist will often employ both agents.

Our modelling approach is closely related to that of Bergemann and Hege (1998, 2002, 2005). They investigate the decision of a venture capitalist who finances a single entrepreneur under uncertainty about the quality of the project and investments needed for its successful realization. Bergemann and Hege (1998) analyze a model in which the quality of a project is not known and has to be resolved through a costly experiment. Their main result is that agency costs lead to inefficiently early stopping of the project. In their paper Bergemann and Hege (2005) extend these results and analyze the difference between relationship financing and arm-length financing. Finally, in the third model Bergemann and Hege (2002) investigate the value of staged financing. The authors show that the use of financing rounds (stages) allows to increase the funding horizon and to make it closer to the socially optimal horizon. We use the framework of Bergemann and Hege to study the patent race in the moral hazard setting.

Another paper which is related to ours is Schmidt (1997). The author studies the incentives of a manager, who operates on the competitive product market. He identifies a "threat-of-liquidation" effect of competition. As the product-market competition increases, the manager is induced to spend more effort, because otherwise the profits of his firm fall below the critical value and the firm will be liquidated. This effect is somewhat similar to ours disciplining effect. In Schmidt (1997), however, competitive environment is exogenously given. We, to the contrary, assume that the venture capitalist can determine the extent of the disciplining effect by employing two agents and specifying in contracts how long each of the competitors will stay in the game.

The structure of this paper is the following. We describe the set-up of the model in Section 2 and derive the sequentially optimal contract in Sections 3 and 4. We introduce strategic interaction among entrepreneurs in Section 5 and discuss the advantage of commitment to finite horizon in Section 6. Section 7 concludes. Proofs and results of numerical simulations can be found in Appendix A.

2 Description of the model

2.1 Innovation process

There are two entrepreneurs with no wealth of their own. Both have an idea (a project) how to solve a particular problem. For example, they try to find a cure against a disease. Following Bergemann and Hege (2002), we assume that the project requires each entrepreneur to complete N sequential stages. These stages are observable and verifiable outcomes, such as a patent, first version of a product, results of markets tests, etc. The stages are sequential in the sense that in order to enter the k-th stage each entrepreneur has to complete successfully previous $(k-1)$ stages. Financing of the projects is done by venture capitalist who provides necessary funds. If all stages are completed, the project generates a prize R and the prize is to be divided between the venture capitalist and the winning entrepreneur. We assume that the winner has a monopoly over the outcome of the project (by patenting the invention), hence the second entrepreneur (the looser of innovation race) does not generate any value. Entrepreneurs and the venture capitalist are risk neutral individuals with common discount rate r.

In order to successfully complete the current stage, the entrepreneur needs to allocate an amount c (provided by the venture capitalist) into the project. In that case the R&D is stochastic and we model the innovation process as a Bernoulli trial, where the stage is completed in the current period with probability p. With probability $1-p$ the entrepreneur does not succeed and needs to invest further (conditional on the fact that his rival has not yet won the race). We assume that the probability of success p is the same for both entrepreneurs. Further, following Lee and Wilde (1980) and Reinganum (1981) we assume that probability of success in each period is independent across the entrepreneurs and across time. Funds are provided by the venture capitalist, but allocation decisions are made by entrepreneurs. They can either invest funds or divert them for private uses. The venture capitalist is not able to observe the allocation decision. All he can observe is a success (completion of the current stage) or an absence of success (which can either mean that an entrepreneur has invested money but failed, or that he has diverted it).[5]

2.2 Moral hazard

We assume that there is a competitive market for innovative projects and a limited supply of venture capital. The venture capitalist can choose any entrepreneur from the pool of identical entrepreneurs. Therefore, the venture capitalist possesses bargaining power, which also means that after paying an entrepreneur the incentive compatible compensation, he retains the residual payoff from the project.

The allocation of funds in this model is subject to a moral hazard: In each period the entrepreneurs face a choice between allocating the funds into R&D and consuming them. The venture capitalist, however, is willing to finance R&D only if he can ensure that funds are allocated truthfully in each period of time. That is, the venture capitalist needs to suggest such reward to both entrepreneurs, so that they prefer to allocate the funds to R&D, rather than to divert them. Moreover, since the allocation of funds is not verifiable, the incentive scheme should reward the entrepreneurs only if a stage was successfully completed.

There are several counteracting forces that determine the size of the incentive payments. On one hand, by consuming funds the entrepreneurs receive the immediate utility c in each period. This way they also ensure themselves further financing, i.e., potential rent of c in the next period. Therefore, in each period of time the venture capitalist should

[5] The innovation process in this game can be interpreted as following. Each entrepreneur owns a coin (representing a project). He tosses the coin and counts a number of “heads” (successes) and “tails” (failures). The first entrepreneur who counts N heads wins the prize R. In order to make one toss, each entrepreneur has to pay a prescribed amount of money c. The venture capitalist provides money for both entrepreneurs in exchange for a share of the prize R.

promise the entrepreneurs a reward which is at least as large as the present value of all investments c which the entrepreneurs can consume. On the other hand, by consuming the funds rather than investing them, each entrepreneur faces a risk that his rival wins the prize. This lowers the expected present value of his future consumption from diverting the funds and therefore limits the option of each entrepreneur to deviate and to consume the funds. Hence, competition might make it cheaper for the venture capitalist to meet the incentive compatibility constraints of the entrepreneurs.

We analyze two scenarios: a basic scenario with entrepreneurs on the same stage of R&D, and a variation with entrepreneurs on different stages of R&D. In the scenario with entrepreneurs on the same stage of R&D, we consider the simplest case, where each entrepreneur needs to complete only one stage in order to complete the whole project. In this scenario we analyze the decision of the venture capitalist whether to employ a single agent or both agents. In the scenario with entrepreneurs on different stages of R&D, we again consider the simplest case, where in order to complete the project the first entrepreneur needs to complete one stage (the *leader*) and the second entrepreneur needs to complete two stages (the *follower*). We are, in particular, interested in the question whether the venture capitalist should in addition to more advanced leader employ the less advanced follower. Bergemann and Hege (2002) analyze a model with a single entrepreneur and N stages. However, for more than one entrepreneur the analysis of the multistage game becomes extremely complicated. In spite of this limitation, our model enables us to illustrate the importance of competition between the entrepreneurs in venture capital financing.

2.3 Definitions and notations

We will call a *regime* (i/j) a situation, where one entrepreneur has i successes (he has successfully completed i stages) and the other entrepreneur has j successes. Both scenarios that we analyze can be then nested within the setup with $N = 2$ (i.e., the project consist of two stages) and $i, j \in \{0, 1\}$: scenario with entrepreneurs on the same stage of R&D corresponds to regime (1/1), and scenario with entrepreneurs different stages of R&D corresponds to regime (1/0).

We will use the following notation:

- T^{ij} denotes the financing horizon in regime (i/j). It this time elapses, the venture capitalist may abandon one or both entrepreneurs.[6]
- V_t^{ij} denotes the value of the project at time t in regime (i/j).
- E_t^L and E_t^F are the expected value of the reward of the leader and the follower respectively at time t in regime (1/0). In regime (1/1) the value function is denoted E_t^{11}.

[6] In some cases, we omit the superscript specifying the regime.

- s_t^L and s_t^F are the rewards, which the leader, respectively the follower, earn upon successful completion of the current stage at time t in regime (1/0). In regime (1/1) the reward is denoted as s_t^{11}.

Furthermore, we will call *regime* (i) a situation, in which the venture capitalist finances only one entrepreneur, who is on i-th stage of R&D. The corresponding value of the project, value function of an entrepreneur, and his reward are denoted as V_t^i, E_t^i, and s_t^i, respectively.

Sometimes, we also denote specific contracts (candidates for the optimal contract) as $\mathcal{C}_k$, where $k = 1, 2, \ldots$. We will then use index k to denote the corresponding financing horizon, value of the project, value function of an entrepreneur, and his reward.

3 Innovation race between identical entrepreneurs

We start with the first scenario, where the venture capitalists faces two identical entrepreneurs, each of which is capable to perform the project. The project consist of a single stage (or equivalently, both entrepreneurs are in the last stage) and the venture capitalist has to decide whether to employ both entrepreneurs, only one of them, or none. It is assumed that the agents observe whether their rival was employed.

In order to give the entrepreneurs incentives to invest in each period of time the venture capitalist has to offer them an appropriate incentive compatible contract.[7] Since the allocation of funds is not observable, the reward, which agents receive according to the contract, has to be conditioned on observable outcomes. Those are the event and the time of success and the identity of the winner.

In regime (i/j) a contract specifies the following terms:

1. Financing horizon T^{ij},

2. Stopping rule that is used when time T^{ij} has elapsed, but no discovery has been made,

3. Rewards for the entrepreneurs s_t^{ij}, depending on the time t when success is achieved, where $0 \leq t \leq T^{ij}$.

Following Bergemann and Hege (2002) and Neher (1999), we assume that the venture capitalist can determine the maximal financing horizon and commit to it. If this horizon have been reached but no success was achieved, then, depending on the stopping rule, either the financing of one entrepreneur or of the whole project will be irrevocably terminated. We justify this assumption in Section 6.

In the situation with identical entrepreneurs we limit our attention to the set of contracts, which use one of the following stopping rules:

[7] Any contract, promising positive expected reward is assumed to satisfy the participation constraint, as the entrepreneurs' outside option is normalized to 0.

1. Stopping rule R_1: Finance both entrepreneurs until one of them wins or until the maximal financing horizon is reached. If neither entrepreneur succeeds, abandon the financing of both.

2. Stopping rule R_2: Finance both entrepreneurs until one of them wins or until the maximal financing horizon is reached. If neither entrepreneur succeeds, abandon one entrepreneur randomly and continue financing in regime (1).

3. Stopping rule R_3: Finance a single entrepreneur until he succeeds but no longer than for T^1 periods.

These rules represent a set of deterministic stopping rules, i.e. they use the observable outcomes to decide, which entrepreneur should be financed further and which should be terminated. The set of deterministic stopping rules is not generally limited to these three stopping rules. There is an additional class of rules within this set, where the principal employs one entrepreneur in period t and the other entrepreneur in period $\tau > t$. We eliminate the latter class of stopping rules on the assumption that an entrepreneur, who is not financed for at least one period, leaves the market (he either becomes an employee or receives financing from other sources, such as bank loan, grant, friends and family, etc.)

To analyze the model we look for sequentially optimal dynamic contract which maximizes the profit of the venture capitalist in each regime of the game. In fact, such contract can be viewed as a sequence of contracts, where the new contract is signed after a regime switches. We require therefore, that in the beginning of the regime (1/1) the venture capitalist *cannot* commit to a contract which will be suboptimal in the regime (1).

In order to find the universally optimal contract we first develop an optimal contract for each of the three stopping rules and then compare the contracts across the stopping rules.

3.1 Value of the venture

The venture capitalist's decision whether to finance one or two entrepreneurs and the choice of the maximal horizon of R&D depends on the expected profit obtained in each case. This profit is the difference between the expected value of the project and the expected compensation of the entrepreneurs.

As our model is formulated in finite time, we can recover the value of the project recursively. Consider first the stopping rule R_3, which corresponds to a case when the principal employs a single entrepreneur - this case is extensively discussed in Bergemann and Hege (2002). In period t the expected value of the project can be written as

$$V_t^1 = Rp + \frac{(1-p)}{1+r} V_{t+1}^1 - c. \tag{1}$$

This value consists of three terms. The last term, c, represents funds which the venture capitalist has to distribute to the entrepreneur. With probability p the entrepreneur makes a discovery in period t. With probability $(1-p)$ the entrepreneur fails, so that the value of the project in period t is the discounted value of the project in period $t+1$, that is $\frac{1}{1+r}V^1_{t+1}$.

The sequence of values is the given by the solution of the difference equation (1) together with terminal condition $V^1_{T+1} = 0$. This condition states that there is no continuation after time T and the value of the project is therefore zero. Following Bergemann and Hege (2002) we consider transition to continuous time. The innovation process becomes a Poisson process and the above difference equation equation becomes a differential equation. Solving it, we obtain the following expression for value function in period t:

$$V^1_t = \frac{(Rp - c)}{r + p}\left(1 - e^{-(r+p)(T-t)}\right). \tag{2}$$

The expression for the value function consists of two factors. The first factor represents the expected payoff from the investment, discounted with a composite discount rate which combines time discount r and the uncertain arrival of success. The second factor shows how the value of the project decreases with time of discovery.

Using the same procedure it is straightforward to derive the value function V^{11}_t for the stopping rule R_1, which corresponds to a case when the principal employs both agents (see Appendix A for details). The recursively determined value of the project in period t is

$$V^{11}_t = Rp(2 - p) + \frac{(1-p)^2}{1+r}V^{11}_{t+1} - 2c \tag{3}$$

This leads to the following value function:

$$V^{11}_t = \frac{2(Rp - c)}{r + 2p}(1 - e^{-(r+2p)(T-t)}).$$

Finally, consider the stopping rule R_2. It dictates that in the case when no entrepreneur succeeds before $t = T$, one of them has to be chosen randomly and financed further for additional number of periods. In this case the value V^{11}_t satisfies equation (4). The expected value of the venture in the terminal period of regime (1/1) is now $V^{11}_{T+1} = V^1_0$, where V^1_0 is determined from (2) for $t = 0$. Solution of the corresponding differential equation leads to the following value function:

$$V^{11}_t = \left(V^1_0 - \frac{2(Rp - c)}{r + 2p}\right) \cdot e^{-(r+2p)(T-t)} + \frac{2(Rp - c)}{r + 2p}. \tag{4}$$

The value functions are summarized in Table 1 in Appendix B.

3.2 Incentives of the entrepreneurs

In each period of time entrepreneurs face a choice between diverting the funds provided by the venture capitalist for private needs, and investing them into the project. In order to motivate entrepreneurs to allocate funds into research and development, the venture capitalist has to promise them a reward which is at least as large as the stream of rent that an entrepreneur can receive diverting the funds.

With our simple model of the R&D process, each entrepreneur has two available strategies: he can either "work" (that is, allocate funds into the project) or "shirk" (that is, divert all funds for private uses). For the time being, we make the assumption that the entrepreneurs do not behave strategically, i.e. each of them believes that the other entrepreneur always "works", or allocates the funds into the project in each period of time. We discuss the strategic interaction in Section 5 and we show that it does not change the results, obtained under the assumption of non-strategic interaction.

In each period of time, the venture capitalist has to offer each entrepreneur such reward, that he finds it incentive compatible to invest in this period, rather than consume funds. For illustration consider the stopping rule R_1. According to this rule, financing of both entrepreneurs is terminated if no success occurred before time T elapses.

The intertemporal incentive compatibility constraint for period t:

$$E_t^{11} = p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)} E_{t+1}^{11} \geq c + \frac{1-p}{1+r} E_{t+1}^{11} \,, \tag{5}$$

with terminal condition $E_{T+1}^{11} = 0$. The left-hand side of (5) represents the expected utility of the entrepreneur, if he allocates the funds into the project at period t. If the entrepreneur wins while his rival loses, which occurs with probability $p(1-p)$, the entrepreneur earns his share s_t^{11}. If there is a tie (i.e., both win, which occurs with probability p^2), he earns this share with probability $\frac{1}{2}$. Last, if nobody wins (with probability $(1-p)^2$), the entrepreneur will receive further financing with present value $\frac{1}{1+r}E_{t+1}^{11}$.

The right-hand side of (5) represents the expected payoff of the entrepreneur from diverting funds at period t. The incentive to divert funds arises from two sources. First, the entrepreneur enjoys the utility c from consuming the funds rather than investing them. Second, by consuming the funds he ensures that financing of the project will continue in the next period with probability $(1-p)$, which is the probability of the rival not making a success. Note that since $(1-p) > (1-p)^2$, by investing the entrepreneur cuts himself off the future stream of rent. If there is only one entrepreneur, as in Bergemann and Hege (2002), then by diverting funds in period t, he guarantees himself that the funding will continue in period $t+1$ with probability 1, unless it is the terminal period. In case of two entrepreneurs, however, funding of each is stochastic and depends on the fact that another entrepreneur has not yet reached success. Therefore, competition softens the incentive compatibility constraint and makes it easier for the venture capitalist to satisfy it.

The venture capitalist aims at paying each entrepreneur the minimal share which will force the latter to invest the funds rather than consume them. To determine the optimal sequence of shares in each time $t = 1, 2, \ldots, T$ the venture capitalist solves the following minimization problem:

$$E_t^{11} = \min_{\{s_t^{11}\}} \quad p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)}E_{t+1}^{11} \tag{6}$$
$$\text{s.t.} \qquad p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)}E_{t+1}^{11} \geq c + \frac{1-p}{1+r}E_{t+1}^{11}.$$

Obviously, in the optimum the incentive compatibility constraint will be binding. Considering the transition to continuous time we derive expressions for the share, which the entrepreneur receives in case of success, and the value function which describes the expected utility of the entrepreneur in each time t, given that he allocates the funds into the project(see Appendix A for the derivation of a value function and of the entrepreneur's share). We obtain

$$s_t^{11} = \frac{c}{p} + E_t^{11}, \tag{7}$$
$$E_t^{11} = \frac{c}{r+p}\left(1 - e^{(r+p)(t-T)}\right). \tag{8}$$

As the entrepreneurs are ex-ante identical, in the sense that they are at the same stage of R&D and have the same probability to complete the project, the value functions (and the shares) are identical for both entrepreneurs.

The compensation scheme, described by the value function E_t^{11}, guarantees that each entrepreneur invests the funds, rather then diverting them, in each period in the regime (1/1). The above expression is very intuitive. The first factor of E_t^{11} represents the value of perpetuity which an entrepreneur would receive if he diverted the funds. The second factor represents a "punishment" for late discovery, in the sense that the share of an entrepreneur decreases over time. Analogically as in the previous case, it is easy to derive the share and the expected utility of the entrepreneurs for the stopping rule R_3:

$$s_t^1 = \frac{c}{p} + E_t^1$$
$$E_t^1 = \frac{c}{r}(1 - e^{-r(T-t)}), \tag{9}$$

Finally, the stopping rule R_2 differs from R_1 again only in the terminal condition. Since there is probability $\frac{1}{2}$ that the entrepreneur will be chosen to continue, we have $E_{T+1}^{11} = \frac{1}{2}E_0^1$, where E_0^1 is given by (9) for $t = 0$, Then it is straightforward to determine the corresponding share and the expected utility of an entrepreneur:

$$s_t^{11} = \frac{c}{p} + E_t^{11},$$
$$E_t^{11} = \left(\frac{1}{2}E_0^1 - \frac{c}{r+p}\right) \cdot e^{-(r+p)(T-t)} + \frac{c}{r+p} \tag{10}$$

The results are summarized in the Table 1 in Appendix B.

3.3 Optimal stopping time

For each stopping rule the venture capitalist maximizes his profit from the project, subject to the incentive compatibility constraints. Given the stopping rule, the choice variables of the venture capitalist are the shares of entrepreneurs and the maximal time horizon. The share is the function of exogenous parameters c, r and p, time t when success is achieved, and the endogenously determined time horizon T. Hence, optimally choosing the financing horizon the principal automatically determines the sequence of shares.

Stopping rule R_1

Consider the stopping rule R_1, which requires that both entrepreneurs are financed until one of them wins or until the financing horizon elapses. The optimal time horizon is derived from the following program:

$$\max_{T\in(0,\infty)} V_0^{11} - 2E_0^{11} \, ,$$

where the value functions V_0^{11} and E_0^{11} are derived in the previous section and are given by (4) and (8) with $t = 0$. The first order condition yields a unique solution to the maximization problem. We will denote the *optimal* financing horizon as T_1^{11}, where

$$T_1^{11} = -\frac{1}{p} \ln \frac{c}{Rp - c} \, .$$

We denote the optimal contract, corresponding to the stopping rule R_1 as $\mathcal{C}_1$. According to this contract the venture capitalist finances both entrepreneurs until one of them succeeds but at most for T_1^{11} periods. In case of success the winner is rewarded with an appropriate share s_t^{11} as given by (7); see also Table 1 in Appendix B for the summary of contract terms.

Stopping rule R_3

The stopping rule R_3 corresponds to the benchmark case with one entrepreneur. It is easy to show that in this case the optimal financing horizon is $T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$ (see also Bergemann and Hege 2002). The resulting contract is denoted $\mathcal{C}_4$.[8] Since the optimal financing horizon depends on costs of R&D and on expected payoff, it is not surprising, that $T^1 = T_1^{11}$. Indeed, two entrepreneurs spend twice as much on R&D, but they also have twice as large probability of success,[9] so that the ratio of R&D costs to the expected payoff remains constant.

Note, that T^1 is positive if and only if $Rp > 2c$. The intuition behind this restriction becomes clear when we re-write inequality as $R > \frac{2c}{p}$. The R&D in our model follows a Poisson process with parameter p, so that the expected time of discovery when a single entrepreneur is employed, is $\frac{1}{p}$. Hence, the requirement $R > \frac{2c}{p}$ means that the venture

[8] Here, we use T^1 instead of T_4^1, since $\mathcal{C}_4$ is the only contract which arises in regime (i).

[9] Intuition for this result is explained in Section 3.4.

capitalist will finance the project only if the value of the prize is larger than the expected cost of R&D plus the expected reward, payed to an agent. Otherwise, it is not profitable for the venture capitalist to finance the project at all. From now on we will assume, that $Rp > 2c$.

Stopping rule R_2

Let us now consider the stopping rule R_2. According to this rule both entrepreneurs will be financed until one of them wins, or until the maximal allowed time elapses. If no success was made, then one entrepreneur will be randomly chosen and financed for additional period of time. For simplicity we denote the expected profit, which the venture capitalist retains as $F(T) = V_0^{11} - 2E_0^{11}$, where the functions V_0^{11} and E_0^{11} are given by (4) and (10) with $t = 0$; see also Table 1 in Appendix B. Maximizing the profit of the venture capitalist, we obtain the following first-order condition:

$$F'(T) = -(r+2p) \cdot B^{11} \cdot e^{-(r+2p)T} + (r+p) \cdot A^{11} \cdot e^{-(r+p)T} = 0,$$
$$\text{where} \qquad A^{11} = E_0^1 - \frac{2c}{r+p}, \qquad B^{11} = V_0^1 - \frac{2(Rp-c)}{r+2p}. \tag{11}$$

Depending on the relation of A^{11} and B^{11} the optimal time can be finite or infinite. First note that B^{11} is always negative. Indeed the inequality $B^{11} < 0$ is equivalent to

$$\frac{Rp-c}{r+p}\left(1 - e^{-T^1(r+p)}\right) < \frac{2(Rp-c)}{r+2p},$$

which obviously holds for all values of parameters $p, r \in (0,1)$ satisfying the feasibility condition $Rp > 2c$.

If $(r+p)A^{11} \leq (r+2p)B^{11} < 0$, then the expected profit $F(T)$ is decreasing in T and the optimal research horizon is zero, so that effectively the venture capitalist employs only one entrepreneur.[10] The resulting contract is identical to the contract $\mathcal{C}_4$.

If $(r+2p)B^{11} < (r+p)A^{11} < 0$, then the optimal research horizon is

$$T_2^{11} = -\frac{1}{p} \ln \frac{r+p}{r+2p} \frac{E_0^1 - \frac{2c}{r+p}}{V_0^1 - \frac{2(Rp-c)}{r+2p}},$$

and the corresponding contract is denoted $\mathcal{C}_2$. According to this contract, the venture capitalist commits to finance both entrepreneurs at most for T_2^{11} periods; if this time elapses without a success, then only one entrepreneur (randomly chosen) will be financed further for the maximum of T^1 periods. The terms of the contract are described in Table 1 in Appendix B.

On the other hand, if $A^{11} \geq 0$, the expected profit $F(T)$ is increasing in T and the optimal research horizon is infinite i.e., the venture capitalist is willing to finance the innovation race infinitely long. The corresponding contract is denoted $\mathcal{C}_3$. This case corresponds to the favorable combinations of low costs of R&D and high probability of success.

[10] Note that $F''(T) < 0$ and $F(0) > 0$.

The condition $A^{11} \geq 0$ directly implies that (in expected terms) the venture capitalist would have to pay higher compensation to one entrepreneur than to two entrepreneurs, i.e., $E_0^1 > \frac{2c}{r+p}$. If this is the case, the venture capitalist always prefers a competitive arrangement to a single entrepreneur.

Remark 1. Note that for all contracts, the value functions and the cost functions at the optimal time are homogeneous of degree 1 in (c, R) and homogeneous of degree 0 in (c, p, r). Therefore, if we denote $W(c, p, r, R)$ the maximal value of the venture capitalist's objective function,[11] then

$$W(c,p,r,R) = R \cdot W\Big(\frac{c}{R}, p, r, 1\Big) = R \cdot W\Big(\frac{\bar{r}c}{Rr}, \frac{\bar{r}p}{r}, \bar{r}, 1\Big), \tag{12}$$

where $\bar{r}$ is some particular value of the discount rate. Hence any comparison of contracts for general values of parameters c, p, r, and R is equivalent to comparison for parameters c and p with an arbitrary value of r and with $R = 1$.[12] Later, without loss of generality, we use the value $\bar{r} = 0.05$ in numerical simulations.

3.4 Optimal contract

For each of the three stopping rules we can now specify a contract in terms of maximum time allowed for research and the share of the prize, which each entrepreneur receives in case of success. As we showed in the previous section for the same stopping rule the optimal contract can take several forms. In any case, the terms of the contracts depend on the probability of success and the normalized costs (that is on the ratio $\frac{c}{R}$; see Remark 1). For each combination of parameters, the venture capitalist will choose among three contracts, corresponding to three stopping rules. The optimal contract then is the one which maximizes the residual payoff of the venture capitalist.

Proposition 1. *Let $Rp > 2c$. Then, in regime* (1/1) *the optimal contract is to finance both identical entrepreneurs for at most $T_1^{11} = -\frac{1}{p} \ln \frac{c}{Rp-c}$ periods and abandon financing of both if no success was made (such contract is denoted $\mathcal{C}_1$).*

The proof of the proposition can be found in Appendix A. The above result is based on two effects: *scale effect* and *disciplining effect.* The former means that two entrepreneurs increase the total probability of success. In particular, when the entrepreneurs' R&D processes are independent (as opposed to Levitt 1995) and are modelled as identically distributed Poisson processes, then the probability of success is exactly doubled.[13]

[11] This is, for example, $V_{0,1}^{11} - 2E_{0,1}^{11}$ at time $T = T_1^{11}$ for contract $\mathcal{C}_1$.

[12] In particular, given c, p, r, and R, we choose an arbitrary $\bar{r} > 0$ and consider new variables $\bar{c} = \frac{\bar{r}c}{Rr}$ and $\bar{p} = \frac{\bar{r}p}{r}$, which gives $W(c, p, r, R) = R \cdot W(\bar{c}, \bar{p}, \bar{r}, 1)$. We rename the variables to c and p by dropping the bar.

[13] In discrete time, the probability that at least one entrepreneur succeeds is in each period equal to $1 - (1 - p^2) = 2p - p^2$. On the other hand, in continuous time, one of the properties of Poisson process claims that the probability of two events (two successes) occurring in time interval $[t, t + \Delta]$ interval converges to zero, as $\Delta \to 0$. Therefore, after transition to continuous time, the second-order terms converge to 0 and the probability that at least one entrepreneur encounters a success in $[t, t + \Delta]$ can be approximated by $2\Delta p$. More precisely, $2\Delta p$ is the first-order approximation of the probability, which can also be written in form $2\Delta p + o(\Delta)$.

Thus, at each moment two entrepreneurs create twice as much value as one entrepreneur. Therefore, when it is profitable to employ a single entrepreneur, then the scale effect makes competition more attractive.

At the same time, the expected reward to be paid to each of the competing entrepreneurs is less than the expected reward of a single entrepreneur:

$$E_1^{11} = \frac{c}{r+p}(1 - e^{-(r+p)(T-t)}) \;<\; \frac{c}{r}(1 - e^{-r(T-t)}) = E^1.$$

Hence, the competition disciplines the entrepreneurs making them working hard for smaller reward, which obviously makes competition more attractive. This effect on incentives we call the *disciplining effect*.

3.5 The effect of competition

Now we compare our result with the first-best case. In the first-best world (without moral hazard) the principal is also always better of employing competing agents rather than a single agent. Indeed, in both cases the reward of the agents is zero. Hence, if the expected value of the project is larger than costs, i.e. $Rp > c$, the venture capitalist is willing to finance a project infinitely long. Therefore, in continuous time the value of the venture with competing agents V_{FB}^{11} and with a single agent V_{FB}^{1} is given by the following functions respectively:[14]

$$V_{FB}^{11} = \frac{2(Rp - c)}{r + 2p}, \qquad V_{FB}^{1} = \frac{Rp - c}{r + p}. \tag{13}$$

Obviously, $V_{FB}^{11} > V_{FB}^{1}$ for any $Rp > c$. This result is due to the scale effect: Two agents succeed two times more often than a single agent. In the situation with the moral hazard the total effect of competition is even more significant due to the effect on the incentives. Our model predicts therefore, that the venture capitalist will always choose to finance competing entrepreneurs, if they are at the same stage of innovation race. This strong conclusion is partially a result of the assumption that the entrepreneurs are considered to be identical. In the next section we show however, that the venture capitalist can often benefit from competition even if the entrepreneurs are situated on the different stages of R&D.

4 Innovation race between the leader and the follower

Consider now the second scenario, where in order to finish the project an entrepreneur has to complete two stages. Assume further that the venture capitalist has in his portfolio an entrepreneur (a *leader*) who has already finished one stage of R&D. Now the venture capitalist faces an opportunity to employ another entrepreneur (a *follower*) who has not yet achieved his first success. Should the venture capitalist finance the innovation race between those two agents or should he rather proceed financing the leader alone?

[14]Subscript FB stands for “first-best.”

When the innovation race starts in a regime with a leader and a follower, the entrepreneurs are not identical from the venture capitalist's point of view. Indeed, the leader has a higher probability of winning a prize. Still, as we discuss in this section, competition can be beneficial, if the presence of the follower considerably limits a rent which the leader can extract from the venture capitalist. The follower has to be a credible threat in a sense that the probability that he makes a breakthrough and wins the race should be sufficiently high. On the other hand, the costs of R&D should be low, compared to the expected prize, so that the duplication of research efforts is justified.

In our notations the game with with a leader and a follower corresponds to regime (1/0). As before, to solve the game we look for sequentially optimal contract that maximizes the profit of the venture capitalist at each regime of the game. That is, we require that the venture capitalist *cannot* draft a contract in regime (1/0) that would be suboptimal in regime (1/1) and/or in regime with a single entrepreneur. Further, we will limit the set of available contracts to the contracts which use one of the following stopping rules:

1. Stopping rule R_1: Finance both entrepreneurs until one of them succeeds or until the maximal financing horizon is reached. If neither entrepreneur succeeds, abandon the financing of both.

2. Stopping rule R_2: Finance both entrepreneurs until one of them succeeds or until the maximal financing horizon is reached. If neither entrepreneur succeeds, stop financing the follower and finance the leader until he succeeds, but no longer than for T^1 periods.

3. Stopping rule R_3: Finance the leader until he succeeds, but not longer than for T^1 periods.

The set of deterministic stopping rules in the race between the leader and the follower is generally not limited to the three stopping rules described above. There is, first of all a class of rules where the venture capitalist employs one agent in period t and another agent in period $\tau > t$. Similar as in Section 3, we eliminate this class of rules on the assumption that an agent who is not financed at least one period leaves the market.

Further, the rules described above favor the leader in a sense that he is always financed at least as long as the follower. Potentially, the venture capitalist could use some stopping rule, which favors the follower. Intuitively, such rules are less attractive for the principal. They unambiguously decrease the probability of success (and hence the expected value of the project) and, while, improving the incentives of the leader, they weaken the incentives of the follower. One example of such stopping rules is the counterpart of rule R_3, where the venture capitalist employs only the follower. Obviously, the expected profit of the principal in the former case is higher. The other possibility is the counterpart of a rule R_2, where the principal abandons the leader and continues financing the follower after the agents fail to deliver a success until the terminal period. It is easy to show, that the tradeoff between stopping rule R_2 and his counterpart is equivalent to the tradeoff between employing only the leader and employing only the follower (the formal argument is provided in the working paper version Goldfayn and Kováč 2005). Indeed, both stopping rules are equivalent in terms of expected value of the project and required incentives in the (1/0) regime. What matters for comparison of two stopping rules is what happens after the terminal period elapses but no success was made. In the first case (rule R_2) the venture capitalist would continue financing the leader. In the second case (counterpart of rule R_2) the venture capitalist would finance the follower, which obviously generates smaller expected profit than the former possibility. Hence, the limitation of the menu of stopping rules to those described above is well justified.

Similarly as before, in order to find the optimal contract, we develop an optimal contract for each stopping rule, and then compare the contracts across stopping rules. A contract specifies, in each regime (i/j) that can be achieved, the following terms:

1. financing horizon T^{ij},
2. stopping rule determining that is used when time T^{ij} has elapsed, but no discovery has been made,
3. rewards for the leader s_t^L, and the follower s_t^F depending on the time t when success is achieved, where $0 \leq t \leq T^{ij}$.

For each stopping rule, the value functions are derived recursively starting from the terminal period of regime (1/0) following the analogical procedure as with identical agents. Therefore, instead of providing all details about the derivation of the value functions, we will only mention the main milestones. In each period t of the game there are the following possibilities:

(a) With probability p the leader wins his second success and the game ends. Note that this also includes the case when both agents complete a current stage.

(b) With probability $p(1-p)$ the follower wins his first success and the game switches to a regime (1/1) (patent race with identical agents).

(c) With probability $(1-p)^2$ neither agent succeeds. If $t < T$ then the game continues further. If $t = T$, then according to the appropriate stopping rule either the project is terminated, or the financing of the follower is terminated and the leader is financed further for at most T^1 periods.

Further notice, that according to the stopping rule R_3 the venture capitalist finances a single agent (the leader). This situation is the same as in regime (1/1) and is analyzed in Section 3. Hence, we will focus on the stopping rules R_1 and R_2. For both stopping rules, the equations, that recursively determine the value of the venture, the expected reward of the leader, and the follower, satisfy:

$$V_t^{10} = pR - 2c + \frac{p(1-p)}{1+r}V_0^{11} + \frac{(1-p)^2}{1+r}V_{t+1}^{10}, \tag{14}$$

$$E_t^L = c + \frac{p}{1+r}E_0^{11} + \frac{1-p}{1+r}E_{t+1}^L, \tag{15}$$

$$E_t^F = c + \frac{1-p}{1+r}E_{t+1}^F. \tag{16}$$

Clearly, the terminal conditions for stopping rule R_1 are $V_{T+1}^{10} = E_{T+1}^{L} = E_{T+1}^{F} = 0$. On the other hand, the terminal conditions for stopping rule R_2 are $V_{T+1}^{10} = V_0^1$, $E_{T+1}^{L} = E_0^1$, and $E_{T+1}^{F} = 0$. Using these conditions and equations (14)–(16) it is straightforward (after transition to continuous time[15]) to derive the expected value of the venture and the expected reward of the agents. The results are summarized in Table 2 in Appendix B.

An important observation is that the reward of the leader has to be higher than the reward of the follower. By diverting funds at some period of time, the leader can guarantee himself a rent $c+\frac{p}{1+r}E_0^{11}$, where $\frac{p}{1+r}E_0^{11}$ is his expected payoff in the case when the follower makes the first success.[16] Therefore, the venture capitalist has to offer the leader an incentive compatible share, i.e., such that the leader's expected reward will be at least as large as the stream of rents $c + \frac{p}{1+r}E_0^{11}$. On the other hand, if the follower consumes the funds in period t, he can only guarantee himself a rent of c in this period. Therefore, his incentive compatible share should be lower than that of the leader.

4.1 Optimal stopping time

Stopping rule R_2

Consider first the stopping rule R_2. In this case the value of the venture and expected reward of the entrepreneurs are described by the following value functions:

$$\begin{aligned} V_0^{10} &= \left(V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}\right)\cdot e^{-(r+2p)T} + \frac{p(R+V_0^{11})-2c}{r+2p}, \\ E_0^L &= E_0^1 \cdot e^{-(r+p)T} + \frac{c+pE_0^{11}}{r+p}(1-e^{-(r+p)T}), \\ E_0^F &= \frac{c}{r+p}(1-e^{-(r+p)T}). \end{aligned}$$

Maximizing the expected surplus of the venture capitalist $G(T) = V_0^{10} - (E_0^L + E_0^F)$ with respect to stopping time T we obtain the first order condition:

$$G'(T) = -(r+2p)B^{10}\cdot e^{-(r+2p)T} + (r+p)A^{10}\cdot e^{-(r+p)T},$$

where

$$A^{10} = E_0^1 - \frac{pE_0^{11}+2c}{r+p}, \qquad B^{10} = V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}. \tag{17}$$

Depending on the relation of A^{10} and B^{10}, the optimal financing horizon can be either zero, positive finite, or infinite. The Lemma 1 summarizes the results; see Appendix A for its proof.

Lemma 1. *Let $Rp > 2c$. Then in regime (1/0) the following statements hold:*

[15] Recall, that due to the property of the Poisson process the probability of two events happening at the same time period is 0. Hence, after transition to continuous time, all terms containing p^2 will become zeros.

[16] In the regime (1/1) the optimal contract is $\mathcal{C}_1$. Henceforth, for ease of notation when referring to the terms of this contract we will relax the index of a contract. That is $E_t^{11} := E_{t,1}^{11}$, $V_t^{11} := V_{t,1}^{11}$, $T^{11} := T_1^{11}$.

(i) *If* $A^{10} > 0$, *then* $B^{10} < 0$. *In that case* $G(T)$ *in monotonically increasing and the optimal stoping time is infinite.*

(ii) *If* $(r+2p)B^{10} < (r+p)A^{10} < 0$, *then function* $G(T)$ *reaches maximum at time*

$$T_6^{10} = -\frac{1}{p}\ln\frac{r+p}{r+2p}\,\frac{E_0^1 - \frac{2c+pE_0^{11}}{r+p}}{V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}},$$

where $T_6^{10} > 0$.

(iii) *If* $A^{10} < 0$ *and* $(r+p)A^{10} < (r+2p)B^{10}$, *then function* $G(T)$ *is monotonically decreasing and the optimal stoping time is zero.*

Note that in case (iii) the venture capitalist finances a single entrepreneur, i.e., $V_0^{10} = V_0^1$ and $E_0^L + E_0^F = E_0^1$. The resulting contract is the same as in regime (1/1) and is again denoted $\mathcal{C}_4$. In case (ii) the venture capitalist finances both entrepreneurs until time T_2^{10} is reached and then abandon the follower and continue financing the leader for additional $T^1 = -\frac{1}{p}\ln\frac{c}{Rp-c}$ periods. The resulting contract is denoted $\mathcal{C}_6$. In case (i), the optimal financing horizon is infinite and the resulting contract is denoted $\mathcal{C}_7$. Detailed conditions and value function for these contracts are described in Table 2 in Appendix B.

Stopping rule R_1

Consider now the stopping rule R_1. Maximizing the surplus of the venture capitalist, it is easy to establish that the optimal stopping time is finite:

$$T_5^{10} = -\frac{1}{p}\ln\frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}.$$

The resulting contract is denoted $\mathcal{C}_5$; see Table 2 in Appendix B for details.

Note that $2c+pE_0^{11} < p(R+V_0^{11})-2c$ is necessary for T_5^{10} to be positive. If the reverse inequality holds, then the optimal stopping time is zero, so that the venture capitalist prefers to finance the leader alone.

Conditions, such as the one above, determine whether a particular contract is feasible. For the contracts with finite stopping time (i.e., contracts $\mathcal{C}_4$, $\mathcal{C}_5$, $\mathcal{C}_6$), these necessary conditions require, that the optimal financing horizon is positive. For contract $\mathcal{C}_7$, the necessary condition requires, that parameters are such, that the optimal financing horizon is infinite. From now on we will call these necessary conditions *feasibility conditions*. We will call a contract *feasible* in the range of parameters, where the corresponding feasibility conditions are satisfied. The range of parameters, where feasibility conditions for each contract $\mathcal{C}_k$, $k = 4,5,6,7$ are satisfied, is shown in Figure 3 in Appendix B.

4.2 Optimal contract

Given the values of parameters, we choose, out of the pool of feasible contracts, the one that maximizes the profit of venture capitalist, i.e., we look for an optimal contract with respect to stopping rules. Investigation of feasibility conditions and optimality of contracts leads to Proposition 2. The proof of the proposition (partly numerical) can be found in Appendix A.

Proposition 2. *Let $Rp > 2c$. Then in regime $(1/0)$ the following statements hold:*

(i) *If $A^{10} > 0$, then the feasible contracts are $\mathcal{C}_4$, $\mathcal{C}_5$ and $\mathcal{C}_7$. The optimal contract is $\mathcal{C}_4$.*

(ii) *If $0 > A^{10}(r+p) > B^{10}(r+2p)$, then the feasible contracts are $\mathcal{C}_4$, $\mathcal{C}_5$ and $\mathcal{C}_6$. The optimal contract is $\mathcal{C}_5$, if parameters are such that $T_5^{10} < T_6^{10} - \frac{1}{r+p} \ln \frac{2c+pE_0^{11} - E_0^1(r+p)}{2c+pE_0^{11}}$. Otherwise, contract $\mathcal{C}_6$ is optimal.*

(iii) *If $A^{10}(r+p) < B^{10}(r+2p)$ and $(2c+E_0^{11}) < p(R+V_0^{11}) - 2c$, then the feasible contracts are $\mathcal{C}_5$ and $\mathcal{C}_4$. The optimal contract is $\mathcal{C}_4$.*

(iv) *If $A^{10}(r+p) < B^{10}(r+2p)$ and $(2c+E_0^{11}) > p(R+V_0^{11}) - 2c$, then the only feasible (hence, the optimal) contract is $\mathcal{C}_4$.*

Let us denote $\mathcal{D}_i$ the domain of parameters (p, c) where contract $\mathcal{C}_i$ is optimal, where $i = 4, 5, 6, 7$. Proposition 2 shows that the domain $\mathcal{D}_7$ is empty and hence the whole parameter space can be divided into three domains $\mathcal{D}_4$, $\mathcal{D}_5$, and $\mathcal{D}_6$, as shown in Figure 1.[17]

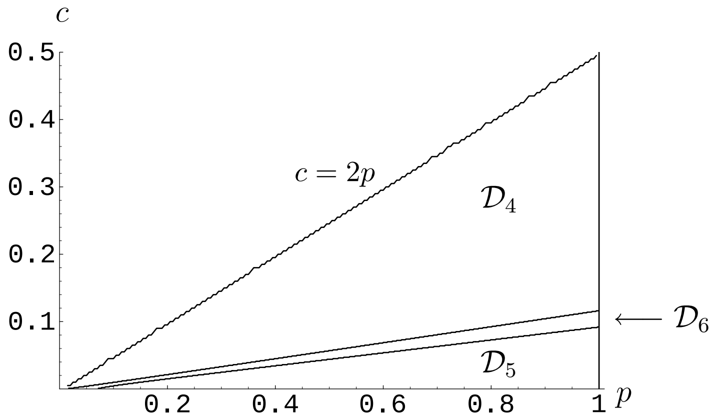

Figure 1: Regime (1/0): Division of the parameter space into three domains according to optimal contracts; for $r = 0.05$

The region $\mathcal{D}_5$ corresponds to the most favorable combination of costs of R&D and the probability of success. In region $\mathcal{D}_4$, on the contrary, for each success probability the costs of R&D are the highest. Finally, in region $\mathcal{D}_6$ the combination of costs and success probability is moderately favorable. It is therefore intuitive that competition is a beneficial

[17] To draw the domains $\mathcal{D}_4$, $\mathcal{D}_5$, and $\mathcal{D}_6$ we considered fixed values of discount rate $r = 0.05$ and prize $R = 1$ and used numerical simulations. On a grid 0.001×0.001 and for values of parameters, such that $p \in [0, 1]$ and $c \in [0, p/2]$ we plotted the points where the constraints for each domain are satisfied. Due to homogeneity of the profit function the choice parameters values is without loss of generality.

arrangement for the venture capitalist, if values of the parameters lie in the domain $\mathcal{D}_5$. In domain $\mathcal{D}_6$ competition is beneficial if the patent race doesn't take too much time. The costs of R&D are, however, relatively high in this domain. Hence, after experimenting with patent race for some time, the venture capitalist will continue financing the leader alone, if the agents fail to deliver a success. Finally, in domain $\mathcal{D}_4$, competition is not beneficial, since the costs are too high to justify the duplication of research efforts.

4.3 The effect of competition

In order to investigate the effect that competition has on the decision to employ competing entrepreneurs, we compare the moral hazard setting with the benchmark case without moral hazard. In the latter case the venture capitalist can perfectly observe the allocation of funds and therefore the incentive compatible reward of both entrepreneurs is zero (this is due to the assumption that the venture capitalist has all bargaining power). Hence, the expected payoff of the venture capitalist equals the expected value of the project. Therefore, for any $Rp > 2c$ the venture capitalist is willing to finance the project infinitely long.

The value of the venture with competing agents V_{FB}^{10} and a single agent (the leader) V_{FB}^{1} are given by the following functions respectively:

$$V_{FB}^{10} = \frac{p(R + V_{FB}^{11}) - 2c}{r + 2p}, \qquad V_{FB}^{1} = \frac{Rp - c}{r + p},$$

where V_{FB}^{11} is given by (13). The venture capitalist will finance only the leader, if $V_{FB}^{1} \geq V_{FB}^{10}$, which is equivalent to the condition:

$$\frac{c}{Rp - c} > \frac{p \cdot r}{(r + 2p)(r + p)}. \tag{18}$$

Otherwise, the venture capitalist will finance both entrepreneurs.

According to condition (18), we divide the parameter space into two domains, as is shown in Figure 2. The border curve between single entrepreneur (SE) and competing entrepreneurs (CE) corresponds to the case of equality.[18] The region above the line represents combinations of costs and success probability, where (18) holds, i.e., where the venture capitalist finances only the leader. If the combination of costs and probability is below the line, then the venture capitalist will prefer to finance both entrepreneurs.

Recall, that in regime (1/1) the venture capitalist prefers to employ competing entrepreneurs, regardless whether the moral hazard is present or not. Without moral hazard this decision is motivated by the *scale* effect: with two entrepreneurs the probability of success is twice as large as with one entrepreneur. With moral hazard there is additional effect of competition, which we call the *disciplining* effect. This effect decreases the rent of each entrepreneur comparing to situation of no competition, so that in case of success the venture capitalist retains larger share of the prize.

[18]The areas were plotted for r is fixed at $r = 0.05$. Due to the homogeneity of the profit functions this choice of parameter is without loss of generality.

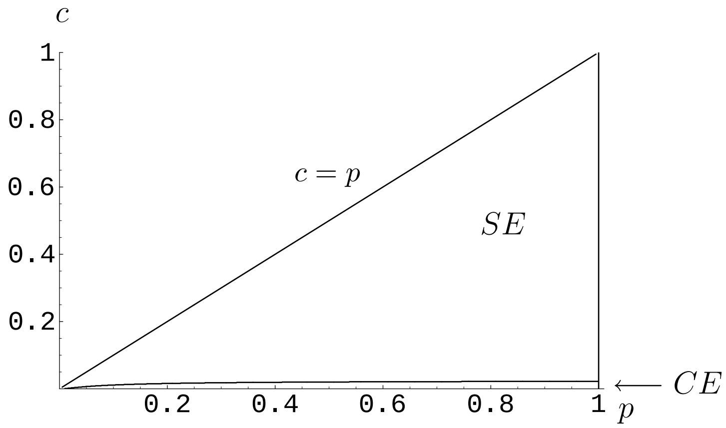

Figure 2: Regime (1/0), no-moral hazard case: Competing entrepreneurs (CE) vs a single entrepreneur (SE); $r = 0.05$

The analysis of a regime with the leader and the follower allows to understand the relative importance of the scale and disciplining effects in the presence of moral hazard. A comparison of Figures 1 and 2 shows that without moral hazard the range of parameters where the competition is beneficial is significantly smaller than in a moral hazard setting (although the whole feasibility region has increased). In the absence of moral hazard the increased probability of success due to competition (scale effect) is almost always not sufficient to justify financing of both the leader and the follower. However, the disciplining effect of competition in case of moral hazard is so important, that the venture capitalist will hire both the leader and the follower, although he does not gain much in terms of success probability. The venture capitalist nevertheless gains from the reduction of rent which he has to pay to both entrepreneurs. For certain combinations of costs and probability, the decrease in compensation of the leader due to competition is large enough to justify the financing of both entrepreneurs (domains $\mathcal{D}_5$ and $\mathcal{D}_6$). Naturally, the competition can be justified only if the follower is not too expensive to finance (c should be relatively small) and the reduction in the rent of the leader due to competition is significant (p should be relatively large).

Another result of competition between entrepreneurs is the increase in the total financing horizon of the project. For the range of costs and probabilities where the venture capitalist chooses to finance both the leader and the follower (i.e., domains $\mathcal{D}_5$ and $\mathcal{D}_6$), the maximal financing horizon is longer with a competitive arrangement than with a single entrepreneur. Indeed, a single entrepreneur (the leader) would be financed for at most of T^1 periods according to contract $\mathcal{C}_4$. If both entrepreneurs are employed, then the maximum financing horizon is $T_k^{10} + T^1 > T^1$, where $k = 5, 6$. Therefore, competition helps to alleviate one of the main problems created by moral hazard — the limitation of the research horizon. We have shown that the first best solution obtained in the absence of moral hazard is to finance the project infinitely long. The same result was obtained for the case of one entrepreneur by Bergemann and Hege (2002). Since the expected value of the project increases in the research horizon, the presence of moral hazard reduces this value. Competition, however, limits the amount of rent which the entrepreneurs can extract from the venture capitalist and hence makes it profitable for the venture capitalist to set a longer financing horizon.

5 Strategic interaction

Up to this point we assumed that the entrepreneurs do not behave strategically, i.e., that each entrepreneur believes that his rival always invests all funds into R&D. In other words, each entrepreneur believes that by diverting the funds in each period, he faces a probability p that his rival wins the prize in the meantime. With this assumption in hand, we have shown that competition softens the incentive compatibility constraints of the entrepreneurs and makes it cheaper for the venture capitalist to provide an incentive compatible reward scheme. As we have discussed, the incentive compatible reward of each entrepreneur is lower in the case of competition, than in the case without competition.

However, if the entrepreneurs are well-trained game theorists and think strategically, they will take into account all possible strategies of the rival. Those can be either "work" (denote it w) or "shirk" (denote it s). Hence, in each period we can model the behavior of the entrepreneurs by a 2×2 game. The venture capitalist, naturally, wants to ensure the (w, w) equilibrium. Otherwise his investments are wasted. Our results already imply that under the compensation schemes considered before, it is optimal for each agent to play w, if his rival plays w (that is, w is the best response to w). In this section we will show these compensation schemes are sufficient to ensure the unique equilibrium (w, w). Note that for this it is sufficient to rule out the equilibrium (s, s).

We start with the situation where two identical entrepreneurs are involved in the innovation race (regime (1/1) in our notations). Consider the terminal period T. Let s_T be the reward of an entrepreneur if he achieves a success. In the table below we summarize payoff of one entrepreneur in four strategic situations (recall that the entrepreneurs are identical).

	w	s
w	$(p - \frac{1}{2}p^2)s_T$	ps_T
s	c	c

In order to ensure that (w, w) is a unique Nash equilibrium (in pure strategies), the reward s_T should be such that: $w \in BR(w)$ and $s \notin BR(s)$, where BR stands for best response.[19] Examining the payoffs, we receive:

$$\begin{aligned} w \in BR(w), &\iff s_T \geq \tfrac{c}{p-\frac{1}{2}p^2} =: s_T^w, \\ s \notin BR(s), &\iff s_T \geq \tfrac{c}{p} =: s_T^s. \end{aligned}$$

Since $s_T^s < s_T^w$ for all $p \in (0,1)$, in the terminal period of the game the venture capitalist can ensure the unique equilibrium (w, w) by promising the entrepreneurs reward $s_T = s_T^w$.

Consider now some period of time $t \leq T-1$ and assume that both entrepreneurs invest in each period $\tau = t+1, \ldots, T$. We will determine such s_t that in period t both entrepreneurs find it incentive compatible to invest funds rather than divert them. The following table shows the payoff matrix of one of the two identical entrepreneur at period t.

	w	s
w	$(p-\frac{1}{2}p^2)s_t + \frac{(1-p)^2}{1+r}E_{t+1}^{11}$	$ps_t + \frac{1-p}{1+r}E_{t+1}^{11}$
s	$c + \frac{1-p}{1+r}E_{t+1}^{11}$	$c + \frac{1}{1+r}E_{t+1}^{11}$

As before, we need to determine s_t such that for each entrepreneur w is a best response to any strategy of a rival:

$$\begin{aligned} w \in BR(w), &\quad \text{iff} \quad s_t \geq \tfrac{2c}{p(2-p)} + \tfrac{2p(1-p)}{p(2-p)(1+r)}E_{t+1}^{11} =: s_t^w, \\ s \notin BR(s), &\quad \text{iff} \quad s_t \geq \tfrac{c}{p} + \tfrac{1}{1+r}E_{t+1}^{11} =: s_t^s. \end{aligned}$$

In order to ensure the unique equilibrium (w, w), the venture capitalist has to promise the entrepreneurs a share $s_t \geq \max\{s_t^s, s_t^w\}$. It can be easily shown that $s_t^w > s_t^s$, since it is equivalent to $\frac{1}{1+r}pE_{t+1}^{11} < c$, which holds since

$$pE_{t+1}^{11} = \frac{pc}{r+p}\left(1 - e^{-(r+p)\left(T^{11}-(t+1)\right)}\right) < \frac{pc}{r+p} < c.$$

Therefore, by promising the entrepreneurs a reward $s_t = s_t^w$ the venture capitalist ensures the equilibrium (w, w). Note, that in continuous time s_t^{11} converges to $\frac{c}{p} + E_t^{11}$, which is exactly the reward we have calculated before, without accounting for strategic interaction.

The result which we have established holds for any $t \leq T-1$, therefore it holds in particular for $t = T-1$. We have proved that in the terminal period the entrepreneurs will invest, if rewarded according to the (w, w) scheme. Therefore, they will also invest in period $(T-1)$ if rewarded according to the (w, w) scheme. Recursively, we can prove that the result holds for any period t of regime (1/1).

It is interesting to observe that if the entrepreneurs are compensated according to the (w, w) scheme, then in regime (1/1) at each period of time the game resembles the

[19] We will assume that when the entrepreneurs are indifferent between strategies “work” and “shirk”, they choose to work.

Prisoners Dilemma game. The entrepreneurs can be better off if they divert the funds simultaneously in all periods. Indeed, in this case the expected payoff of each entrepreneur is $\frac{c}{r}(1-e^{-rT})$, i.e., a properly discounted stream of rent c. If both entrepreneurs invest, then the expected reward of each is $\frac{c}{r+p}(1-e^{-(r+p)T}) < \frac{c}{r}(1-e^{-rT})$. But under the incentive scheme (w, w), "work" is always the best response to "shirk", therefore, a potentially attractive (for entrepreneurs) situation (s, s) is not a subgame perfect Nash equilibrium.

Following very similar lines of reasoning, it is easy to prove that in regime (1/0), rewarding entrepreneurs according to (w, w) scheme ensures the unique equilibrium (w, w). The following discussion applies both to contract $\mathcal{C}_5$ and contract $\mathcal{C}_6$. Consider the terminal period T. The matrix below summarizes the payoff of the follower (the leader is the row player and the follower is the column player).

	w	s
w	$p(1-p)(s_T^F + \frac{1}{1+r}E_0^{11})$	c
s	$p(s_T^F + \frac{1}{1+r}E_0^{11})$	c

Investigating the payoff of the follower, we derive the following conditions:

$$\begin{aligned} w \in BR(w), \quad &\iff \quad s_T^F \geq \tfrac{c}{p(1-p)} - \tfrac{1}{1+r}E_0^{11} =: s_T^{F,w}, \\ s \notin BR(s), \quad &\iff \quad s_T^F \geq \tfrac{c}{p} - \tfrac{1}{1+r}E_0^{11} =: s_T^{F,s}. \end{aligned}$$

Comparing the compensation of the follower in case when the leader works with his compensation in case when the leader shirks, we establish that $s_T^{F,w}$ is always larger than $s_T^{F,s}$. Therefore, if the venture capitalist promises the follower a reward $s_T^F = s_T^{F,w}$ he ensures that the follower will invest irrespective of a strategy of the leader. This rules out equilibrium (s, s). Hence, to enforce the (w, w) equilibrium it is enough to compensate the leader so that his best response to "work" is "work". The same logic holds for any period $t < T$. Therefore, if both entrepreneurs are compensated as if the other always invests (i.e., "work"), the unique equilibrium of the game is (w, w). This justifies our approach in Sections 3 and 4.

6 Finite horizon and commitment to stop

So far, we have assumed that the venture capitalist can choose the financing horizon for each regime and can commit to it. This means that if the maximum time allowed for experimentation in regime (i/j) elapses without success, then depending on terms of the contract either the project will be irrevocably abandoned, or the venture capitalist will abandon financing of follower. In this section we provide a rationale for that assumption.

If we assume that the venture capitalist cannot commit to stop the project after the maximal allowed time has elapsed, then he will finance the entrepreneurs infinitely long.

Suppose that in regime (i/j) the contract between the venture capitalist and entrepreneurs determines some (optimal) time T^{ij}. If this time elapses but no success was made by any entrepreneur, the venture capitalist is willing to start the game from the beginning, as if the world is in the first period of regime (i/j). Indeed, all costs that the venture capitalists has incurred up to time T^{ij} are sunk, and the game has not changed since the venture capitalist made his optimal decision at $t = 0$ of regime (i/j). Because of this feature of our model (sunk costs and independent probability of success in each period), the venture capitalist is willing to finance the entrepreneurs infinitely long, if he enters the game once.

If the venture capitalist cannot commit to stopping the project, he is also not able to condition further financing on successful completion of predetermined stages or benchmarks. In a world, where commitment is not credible, the venture capitalists will finance entrepreneurs until one of them wins the prize.

However, empirical literature on venture capital documents, that stage financing, which is conditional on successful completion of prescribed milestones, is one of the most important and commonly used control mechanisms in venture capital financing.[20] Therefore, the commitment assumption is not only realistic, but is essential for the ability of the venture capitalist to include the provision about the milestones into the contract.

Obviously, in our model the venture capitalist prefers to commit to finite financing horizon. Commitment to stop financing of the project is an important punishment mechanisms, that allows to decrease compensation of the entrepreneurs and therefore to increase profits of the venture capitalist, comparing to a situation with no commitment. In the model, however, there is no endogenous mechanism, which would make the ex-ante commitment credible ex-post. Hence, to justify the commitment power of the venture capitalist in our model, we make an assumption, that the venture capitalist is wealth-constrained.

This assumption is well supported by the evidence about practice of the venture-capital funds. According to Inderst and Munnich (2003), the venture capital funds are normally close-ended, which means that funds are raised once from the investors and are directed afterwards into the portfolio of projects. The partnership agreements, which govern the venture capital funds, often contain a covenant that limits a possibility of the venture capitalist to raise further investments. Likewise, the partnership agreements restrict ability of the venture capitalist to transfer investments across projects and across different funds, run by the same partners. The wealth-constrained venture capitalist can credibly commit to limit resources directed to each of his portfolio projects and hence can commit to the finite financing horizon.

In the world described in our model, this commitment can be understood as the following. Ex ante, the venture capitalist is able to calculate the optimal period of time, during which he is willing to finance the project. He then commits a corresponding amount of money for this project and commits all other resources to his other portfolio projects. The partnership agreements restrict the ability of the venture capitalist to raise additional funds and, therefore, the commitment to stop the project is credible.[21]

[20] See, for example, Kaplan and Stromberg (2003) and Sahlman (1990).

[21] More realistic approach to model the venture capital process is to assume the venture capitalist have some prior believes about quality of the project. If the project fails to succeed, the venture capitalist

7 Conclusion

In this paper we study innovation race in the moral hazard setting. We explore a model where two entrepreneurs simultaneously develop a project which, if successful, generates a fixed prize R. The project is developed in stages and the first entrepreneur who completes the second stage wins the prize. Research and development is financed by the venture capitalist, but the funds are allocated by the entrepreneurs. This creates a moral hazard problem: the entrepreneurs can divert the funds to their own uses. We investigate two possible scenarios: a basic scenario where both entrepreneurs are at the same (the last) stage of R&D, and its variation where one of the entrepreneurs is a leader and another is a follower.

We identify two effects which make the financing of competing entrepreneurs beneficial for the venture capitalist. First effect is the higher probability of success (scale effect) and the second is less obvious effect which competition has on incentives (disciplining effect). In order to highlight the importance of competition in the moral hazard setting, we compare it with the benchmark setting without moral hazard. The analysis reveals, that in the scenario where both entrepreneurs are on the same stage of R&D both effects are important. Due to the scale effect financing of competing entrepreneurs is attractive in the absence of moral hazard. With moral hazard, the disciplining effect reinforces the scale effect making the financing of competing entrepreneurs even more attractive.

However, in the scenario with a leader and a follower, the scale effect is of little importance, so that without moral hazard the follower will almost never be employed. Nevertheless, with moral hazard in place, the presence of a competitor allows to reduce significantly the rent of the leader, which makes competition a beneficial arrangement for the large range of parameters. We also find that by improving incentives the competition allows the venture capitalist to increase the maximal research horizon during which he is willing to finance the project, making it therefore closer to the first-best (infinite) horizon. The prediction that the projects are financed longer in the presence of competing ventures provides a scope for the empirical test of the model since many databases on venture capital contain information about number and duration of rounds.

Our key finding is that competition can be used by the venture capitalist as an effective cure against the moral hazard, in a situation where the allocation of funds by the entrepreneurs is not observable. Hence, competition serves as a "natural" mechanism that allows to improve the efficiency of research and development. The existence of such mechanism is particulary important in those cases, where the use of complicated security schemes, developed in the venture capital literature, is difficult or not possible at all.

In terms of empirical implications this result suggests that in particular grant agencies and government agencies, as well as venture capital firms that are active on the less developed capital markets, should use competition between portfolio projects as a mechanism of mitigating the agency problems. Indeed, the former, usually do not use complicated

becomes pessimistic and will eventually abandon the project. This mechanism ensures that the project will be stopped in finite time and is extensively discussed in Bergemann and Hege (1998, 2005).

security schemes either due to the lack of expertise or following the established practices.[22] The latter, on the other hand, might find it difficult to use such schemes on the capital market which is not sufficiently developed to accommodate them.

The idea that competition positively affects incentives of the agents (i.e., relaxes their incentive constraint) has already been applied in various settings, like yardstick competition (Tirole 1997, pp. 41–42) or design of team incentives (Holmstrom 1982). We contribute to this literature by investigating the effect of competition on incentives in the dynamic framework, where only the winner's output matters to a principal. We show, that in this framework, competition has a positive incentive effect even when the research technologies are independent (which is not the case in yardstick competition).

Alternative approach to modelling a patent race in the moral hazard framework is to consider a setup where the entrepreneurs have different probability of success, in a sense that the same stage of R&D corresponds to identical probabilities of success, and a different stage of R&D corresponds to asymmetric probability of success. Then our results suggest that as asymmetry between entrepreneurs increases, the positive effect of competition becomes less pronounced. Moreover, in a setting with asymmetric entrepreneurs, the competition is beneficial if the value of the output relative to the costs is high and there is a high chance that the asymmetry will be eliminated (namely probability of success is relatively high). In our future research we aim at investigating this alternative approach in more details.

[22]Consider for example the practise of NIH. The grant is usually split into several budget periods (analogy of stages in our model). Within each stage the financing is provided on a cash request basis, where the cash is transferred to the grantee's account based on his need. See `grants.nih.gov/grants/managing_awards.htm`

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A Appendix: Proofs

Derivation of the value functions in Section 3.1. For illustration consider regime (1/1) and stopping rule R_1. When Δ is the length of the time period, Equation 3 can be rewritten as:

$$V_t^{11} = 2(R\Delta p - \Delta c) - R(\Delta p)^2 + \frac{1 - 2\Delta p + (\Delta p)^2}{1 + \Delta r} V_{t+\Delta}^{11},$$

Dividing the expression by Δ and taking $\Delta \to 0$ we receive the following differential equation:

$$V_t^{11}(r + 2p) = 2(Rp - c) + \dot{V}_t^{11}.$$

Solving the differential equation with terminal condition, which translates to $V_T^{11} = 0$, we receive the expression for V_t^{11}:

$$V_t^{11} = \frac{2(Rp - c)}{r + 2p}(1 - e^{-(r+2p)(T-t)}).$$

The derivation of the value function for the case, where stopping rule R_2 applies is identical except for the boundary condition, which now translates to $V_T^{11} = V_0^1$ in continuous time. Recall that V_0^1 is the value of the project in regime (1), i.e., when only a single entrepreneur is employed.

To derive value functions of entrepreneurs E_t^{11} and their incentive compatible shares s_t^{11} we use the same approach. Consider again regime (1/1), stopping rule R_1. The minimization program, which allows us to determine the optimal share s_t and expected reward E_t^{11} of the entrepreneur is given in Section 3 by problem (6). With incentive compatibility constraint being binding this problem results in the following expression for a share s_t^{11}:

$$ps_t - \frac{1}{2}p^2 s_t = c + \frac{p(1-p)}{1+r} E_{t+1}^{11}. \tag{19}$$

Considering transition of equality (19) to continuous time, we receive:

$$s_t = \frac{c}{p} + E_t^{11}$$

Since incentive compatibility constraint is binding in equilibrium, we can derive solution to the minimization problem (6) from the following equality:

$$E_t^{11} = c + \frac{1-p}{1+r} E_{t+1}^{11}. \tag{20}$$

Considering transition of the equation (20) to continuous time we obtain again a differential equation. After solving the differential equation, with the terminal condition which translates to $E_T^{11} = 0$, we receive the expression for the value function of an entrepreneur:

$$E_t^{11} = \frac{c}{r+p}\left(1 - e^{(r+p)(t-T)}\right).$$

□

Proof of Proposition 1. The proof is divided into two parts depending on the sign of A^{11}. If the parameters are such that $A^{11} > 0$, then the feasible contracts are $\mathcal{C}_1$, $\mathcal{C}_3$ and $\mathcal{C}_4$. On the other hand, if $A_{11} \leq 0$, then the available contracts are $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_4$. We will show that in both cases contract $\mathcal{C}_1$ is optimal.

First we show that contract $\mathcal{C}_1$ is always (regardless of A^{11}) preferred to contract $\mathcal{C}_4$. Translated into profits, this is equivalent to the inequality $V_{0,1}^{11} - 2E_{0,1}^{11} > V_0^1 - E_0^1$, with $V_{0,1}^{11}$, $E_{0,1}^{11}$, V_0^1, and E_0^1 given in Table 1. After substitution, this can be rewritten as

$$\frac{2(Rp-c)}{r+2p} - \frac{Rp+c}{r+p} + \frac{c}{r} - \left(\frac{2(Rp-c)}{r+2p}e^{-2pT} - \frac{Rp+c}{r+p}e^{-pT} + \frac{c}{r}\right)e^{-rT} > 0.$$

Note that the optimal stopping time T is the same for both contracts is $T^1 = -\frac{1}{p}\ln\frac{c}{Rp-c}$. Therefore, $e^{-pT} = \frac{c}{Rp-c}$. Using a substitution

$$x = \frac{c}{Rp-c}, \tag{21}$$

or equivalently $c = Rp\frac{x}{1+x}$, we the rewrite the above inequality as

$$\frac{Rp}{1+x}\left[\frac{2}{r+2p} - \frac{1+2x}{r+p} + \frac{x}{r} - \left(\frac{2x^2}{r+2p} - \frac{x(1+2x)}{r+p} + \frac{x}{r}\right)x^{r/p}\right] > 0.$$

Note that $e^{-pT} = x$ and the assumption $Rp > 2c > 0$ implies that $x \in (0,1)$. Multiplying the last inequality by $(r+2p)(r+p)r(1+x)/(Rp)$ yields

$$r^2 + (p-r)(2p+r)x + p(2rx - 2p - r)x^{1+r/p} > 0.$$

Denote the left-hand side of this inequality as $f(x)$.[23] Then

$$\begin{aligned} f'(x) &= (r+2p)[2rx^{1+r/p} - (p+r)x^{r/p} + (p-r)], \\ f''(x) &= (r+2p)\left[2r\left(1+\frac{r}{p}\right)x^{r/p} - (p+r)\frac{r}{p}x^{-1+r/p}\right]. \end{aligned}$$

First observe that $f(0) = r^2 > 0$, $f(1) = 0$, $f'(1) = 0$, $f''(1) = r(r+p)(r+2p)/p > 0$. Moreover, for $p \le r$, the function f is decreasing on interval $(0,1)$, since $f'(x) < (r+2p)[2rx^{r/p} - (p+r)x^{r/p} + (p-r)] = (r+2p)(p-r)(1-x^{r/p}) < 0$. Hence, $f(x) > f(1) = 0$, for $p \le r$.

On the other hand, for $p > r$ we have $f'(0) = (r+2p)(p-r) > 0$. Therefore, $f(x) > f(0)$ in some neighborhood of 0. Now, assume by contradiction that $f(x_0) = 0$ for some $x_0 \in (0,1)$. Then by continuity there exists some $x_1 \in (0, x_0)$ such that $f(x_1) = f(0)$, which (according to *Rolle's theorem*) implies that there exist some $x_2 \in (0, x_1)$ and $x_3 \in (x_0, 1)$ such that $f'(x_2) = f'(x_3) = 0 = f'(1)$. Therefore, the equation $f''(x) = 0$ has at least two solutions in interval $(0,1)$, which is a contradiction, since $f''(x) = 0$ only if $x = \frac{1}{2}$. This proves that contract $\mathcal{C}_1$ is preferred to contract $\mathcal{C}_4$.

Now, we will show that for $A^{11} > 0$, contract $\mathcal{C}_1$ is preferred to $\mathcal{C}_3$. Obviously the latter contract is a limiting case of the former, when the research horizon is infinity. However, for contract $\mathcal{C}_1$ the optimal time $T_1^{11} = -\frac{1}{p}\ln\frac{c}{Rp-c}$ is finite. Hence, contract $\mathcal{C}_1$ with research horizon T_1^{11} is more profitable for the venture capitalist than contract $\mathcal{C}_1$ with any other research horizon, including infinite research horizon.[24] Therefore, contract $\mathcal{C}_1$ is better than contract $\mathcal{C}_3$.

[23] Note that f is $\mathcal{C}^2$ on $(0,1]$.

[24] One can easily see that for the stopping rule R_1: $\frac{\mathrm{d}}{\mathrm{d}T}(V_0^{11} - 2E_0^{11}) < 0$, when $T > T_1^{11}$.

It remains to prove that contract $\mathcal{C}_1$ is preferred to contract $\mathcal{C}_2$, i.e., that $V_{0,1}^{11} - 2E_{0,1}^{11} > V_{0,2}^{11} - 2E_{0,2}^{11} > 0$, with $V_{0,1}^{11}$, $E_{0,1}^{11}$, $V_{0,2}^{11}$, and $E_{0,2}^{11}$ given in Table 1. This can be rewritten as follows:

$$
\begin{aligned}
&-\frac{2(Rp-c)}{r+2p}e^{-(r+2p)T_1^{11}} + \frac{2c}{r+p}e^{-(r+p)T_1^{11}} - \\
&-\left(V_0^1 - \frac{2(Rp-c)}{r+2p}\right)e^{-(r+2p)T_1^{11}} + \left(E_0^1 - \frac{2c}{r+p}\right)e^{-(r+p)T_1^{11}} > 0.
\end{aligned}
$$

Using again the substitution (21), we obtain

$$
e^{-pT_1^{11}} = x, \qquad \text{and} \qquad e^{-pT_2^{11}} = \frac{x(p+r)}{r}\cdot\frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}.
$$

Then, the above inequality can be, after multiplying by $r(p+r)(2p+r)(1+x)/(Rpx^{2+r/p})$, rewritten as follows:[25]

$$
2r - \left[r-p+(r+p)x^{r/p}\right]\left[\frac{p+r}{r}\cdot\frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}\right]^{1+r/p} > 0.
$$

Similarly as in the first part of this proof, denote the left-hand side of this inequality as $g(x)$. Observe that $g(1) = 0$ and that

$$
\begin{aligned}
g'(x) = \ & \frac{(r+p)(r+2p)}{p^2}\left[\frac{p+r}{r}\cdot\frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}\right]^{1+r/p} x^{-1+r/p} \times \\
& \times \frac{r^2(x-1)+p^2x(x^{r/p}-1)}{r+(r+2p)x^{1+r/p}},
\end{aligned}
$$

which is negative, since $0 < x < 1$. Therefore, $g(x) > g(1) = 0$ for all $x \in [0,1)$, which completes the proof. □

Proof of Lemma 1. 1. Using the expressions for V_0^{11}, E_0^{11}, V_0^1, and E_0^1 from Table 1 in Appendix B, we obtain

$$
\begin{aligned}
A^{10} &= \frac{c}{r(p+r)^2}\big(p^2 - pr - r^2 - (r+p)^2e^{-rT} + pre^{-(r+p)T}\big), \\
B^{10} &= \frac{1}{(p+r)(r+2p)^2}\big[c(r+p)(r+2p) - pr(Rp-c) + \\
&\quad +(Rp-c)\big(-(r+2p)^2e^{-(r+p)T} + 2p(r+p)e^{-(r+2p)T}\big)\big],
\end{aligned}
$$

with T being the optimal stopping time for contracts $\mathcal{C}_1$ and $\mathcal{C}_4$ from regime (1/1), which is the same, i.e., $T = T^1 = T_1^{11} = -\frac{1}{p}\log\frac{c}{Rp-c}$.

Similarly as in the Proof of Proposition 1 we use the substitution (21), or $c = Rp\frac{x}{1+x}$. In addition, to simplify the expressions, we use another substitution

$$
z = \frac{r}{p},
$$

[25] Note that $r-p+(r+p)x^{r/p} > 0$, since $A^{11} = -Rpx/[r(r+p)(1+x)]\cdot[r-p+(r+p)x^{r/p}]$.

or $r = zp$. Given the conditions on parameters, we have $x \in (0,1)$ and $z > 0$. With this substitution, e^{-rT} simplifies to a nice form x^z and the above expressions can be rewritten as follows:

$$\begin{aligned} A^{10} &= \frac{x[1 - z - z^2 - (1+z)^2 x^z + z x^{1+z}]}{(1+x)z(1+z)^2}, \\ B^{10} &= \frac{-z + (1+z)(2+z)x - (2+z)^2 x^{1+z} + 2(1+z)x^{2+z}}{(1+x)(1+z)(2+z)^2}. \end{aligned}$$

For simplicity denote $a(x)$ and $b(x)$ the numerators of A^{10} and B^{10}, respectively. Note that since their denominators are positive, the signs of A^{10} and B^{10} are the same as the signs of $a(x)$ and $b(x)$, respectively.

Depending on the sign of $1 - z - z^2$, we discuss two cases. First, when $1 - z - z^2 \le 0$, then $a(x) < 0$, since $-(1+z)^2 x^z + z x^{1+z} = [-(1+z+z^2) - z(1-x)]x^z < 0$. Second, when the inequality $1 - z - z^2 > 0$ holds, we will prove a stronger statement that this inequality already implies $b(x) < 0$, regardless of the sign of $a(x)$. Note that for $z > 0$, the condition $1 - z - z^2 > 0$ is equivalent to $0 < z < \frac{1}{2}(\sqrt{5} - 1) \approx 0.6180$. Obviously $b(0) = -z$ and $b(1) = 0$. Taking the derivatives of $b(x)$ we obtain

$$\begin{aligned} b'(x) &= (1+z)(2+z)\big[1 - \big(z + 2(1-x)\big)x^z\big], \\ b''(x) &= (1+z)(2+z)x^{-1+z}[2(1+z)x - z(2+z)]. \end{aligned}$$

Then $b'(0) = (1+z)(2+z) > 0$ and $b'(1) = (1+z)(2+z)(1-z) > 0$. The second derivative implies that b is concave in the interval $(0, x_1)$ and convex on $(x_1, 1)$, where $x_1 = \frac{z(2+z)}{2(1+z)} < \frac{1}{2}$, due to assumption $1 - z - z^2 > 0$. Therefore, b has a local maximum (denote it x_2) on interval $(0, x_1)$ and a local minimum on $(x_1, 1)$. Its possible shape is illustrated on Figure 4 in Appendix B. Hence, in order to prove that $b(x) < 0$ on $(0,1)$ it remains to show that $b(x_2) < 0$. Although it is not possible to find a closed formula for x_2, we know that

$$x_2^z = \frac{1}{z + 2(1 - x_2)}.$$

Using this, we obtain

$$\begin{aligned} & 2[z + 2(1-x_2)]b(x_2) = \\ &\quad = 2[-z + x_2(1+z)(2+z)]\,[z + 2(1-x_2)] - \\ &\quad\quad -2(2+z)^2 x_2 + 4(1+z)x_2^2 = \\ &\quad = -4(1+z)^2 x_2^2 + z(z^2 + 4z + 6)x_2 - z(2+z) = \\ &\quad = -[2(1+z)x_2 - z(2+z)]^2 + z[z + 2(1-x_2)](z^2 + 2z - 2) < \\ &\quad < 2z[z + 2(1-x_2)](z^2 + z - 1) < 0. \end{aligned}$$

As a consequence, $A^{10} > 0$ implies that $G'(T) > 0$ for all $T \ge 0$. Hence the optimal stoping time is infinite.

2. The optimality condition $G(T) = 0$ can be rewritten as $e^{-pT} = \frac{(r+p)A^{10}}{(r+2p)B^{10}}$. The condition $(r+2p)B^{10} < (r+p)A^{10} < 0$ implies that $e^{-pT_2^{10}} \in (0,1)$, i.e., T_2^{10} is positive and finite. Moreover, we have $G''(T) = (r+2p)^2 B^{10} e^{-(r+2p)T} - (r+p)^2 A^{10} e^{-(r+p)T}$, which yields $G''(T_2^{10}) = (r+p)pA^{10} e^{-(r+p)T} < 0$.

3. We consider two cases. If $B^{10} \geq 0$, then obviously $G'(T) < 0$. If $B^{10} < 0$, then $G'(T) < [-(r+2p)B^{10} + (r+p)A^{10}]e^{-(r+p)T} < 0$ for all $T \geq 0$. Hence, $G(T)$ is monotonically decreasing and the optimal stoping time is zero.

□

Proof and numerical simulations for Proposition 2.

1. Contract $\mathcal{C}_4$ is feasible whenever $Rp > 2c$. If the second stopping rule is applied, the optimal stopping time is infinity (see discussion in Section 4) and contract $\mathcal{C}_7$ is feasible.

   The conditions $A^{10} > 0$ and $B^{10} < 0$ imply that $pE_0^{11} + 2c < (r+p)E_0^1$ and $(r+2p)V_0^1 < p(R+V_0^{11}) - 2c$ respectively. Moreover, from $V_0^1 > E_0^1 > 0$ we get $(r+p)E_0^1 < (r+2p)V_0^1$. Combining the inequalities, we obtain that

   $$\frac{pE_0^{11} + 2c}{r+p} < \frac{p(R+V_0^{11}) - 2c}{r+2p}, \quad \text{and hence} \quad T_1^{10} = -\frac{1}{p}\ln\frac{2c + pE_0^{11}}{p(R+V_0^{11}) - 2c} > 0,$$

   which means that contract $\mathcal{C}_5$ is feasible. We have proved that if the feasibility condition $A^{10} > 0$ is satisfied, then the pool of available contracts is $\mathcal{C}_4$, $\mathcal{C}_5$, $\mathcal{C}_7$. Further we will compare the surplus which the venture capitalist retains with each contract, in order to choose the optimal one.

   Consider contracts $\mathcal{C}_4$ and $\mathcal{C}_7$. From the Proof of Lemma 1 we know that contract for $A^{10} > 0$, the contract $\mathcal{C}_7$ is optimal among all contracts with stoping rule R_2. As contract $\mathcal{C}_4$ is a degenerate case of this stoping rule (when the research horizon is zero), condition $A^{10} > 0$ then implies that $\mathcal{C}_7 \succ C_4$.[26]

   Further, let us compare contract $\mathcal{C}_5$ and contract $\mathcal{C}_7$. In case of contract $\mathcal{C}_5$ the surplus of the venture capitalist is maximized at finite stopping time, $T_5^{10} = -\frac{1}{p}\ln\frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}$. However, if the financing horizon is infinite, then $\mathcal{C}_5$ is identical to contract $\mathcal{C}_7$. Hence, the former contract is always preferred to the latter.

   In summary we get $\mathcal{C}_5 \succ C_7 \succ C_4$. Hence, the optimal contract is $\mathcal{C}_5$. Note that condition $A^{10} > 0$ implies that $E_0^1 > E_{0,5}^F + E_{0,5}^L$. In other words, competing entrepreneurs together require less compensation, than would a single entrepreneur.

2. Assume that $0 > A^{10}(r+p) > B^{10}(r+2p)$. According to Lemma 1, contract $\mathcal{C}_6$ is feasible. Recall, that

   $$A^{10} = E_0^1 - \frac{pE_0^{11} + 2c}{r+p}, \qquad B^{10} = V_0^1 - \frac{p(R+V^{11}) - 2c}{r+2p}.$$

   Hence, the inequality $A^{10}(r+p) > B^{10}(r+2p)$ implies that

   $$0 < (pE_0^{11} + 2c) - E_0^1(r+p) < [p(R+V_0^{11}) - 2c] - V_0^1(r+2p).$$

   Since $E_0^1(r+p) < V_0^1(r+2p)$, it necessarily must be that $pE_0^{11} + 2c < p(R+V_0^{11}) - 2c$. Hence, $T_1^{10} > 0$ and contract $\mathcal{C}_5$ is feasible as well. Therefore, the pool of contracts consists of $\mathcal{C}_5$, $\mathcal{C}_6$ and $\mathcal{C}_4$.

[26]The relation "$\succ$" is used to denote preferences between contracts from the viewpoint of the venture capitalists, i.e., that one contract generates a larger profit for the venture capitalist than another one.

Let us first compare contracts $\mathcal{C}_5$ and $\mathcal{C}_6$. The former contract is preferred to the latter, if and only if

$$V_{0,5}^{10} - V_{0,6}^{10} > \left(E_{0,5}^{(10),L} + E_{0,5}^{(10),F}\right) - \left(E_{0,6}^{(10),L} + E_{0,6}^{(10),F}\right), \tag{22}$$

where all value functions are given in Table 2 in Appendix B. After straightforward but tedious calculations we conclude that inequality (22) is equivalent to

$$T_5^{10} < T_6^{10} - \frac{1}{r+p} \ln \frac{2c + pE_0^{11} - E_0^1(r+p)}{2c + pE_0^{11}}.$$

In that case, contract $\mathcal{C}_5$ is optimal. Otherwise, the optimal contract is $\mathcal{C}_6$.

Note, that now it is sufficient to prove, that $\mathcal{C}_6$ is preferred to contract $\mathcal{C}_4$, always when the feasibility condition $0 > A^{10}(r+p) > B^{10}(r+2p)$ holds. If this is the case, then $\mathcal{C}_5$ will be optimal, when $\mathcal{C}_5 \succ C_6 \succ C_4$ and $\mathcal{C}_6$ will be optimal, when $\mathcal{C}_6 \succ C_5$ and $\mathcal{C}_6 \succ C_4$.

Contract $\mathcal{C}_6$ is better, than contract $\mathcal{C}_4$, if and only if the following inequality holds:

$$V_0^1 e^{-(r+2p)T} + \frac{p(1+V_0^{11}) - 2c}{r+2p}\left(1 - e^{-(r+2p)T}\right) -$$
$$-E_0^1 e^{-(r+p)T} - \frac{2c + pE_0^{11}}{r+p}\left(1 - e^{-(r+p)T}\right) > V_0^1 - E_0^1,$$

This can be re-written in the form

$$A^{10}\left(1 - e^{-(r+p)T}\right) > B^{10}\left(1 - e^{-(r+2p)T}\right), \tag{23}$$

where A^{10} and B^{10} are defined above. Consider now two cases:

(a) If $0 > A^{10} > B^{10}$, then inequality (23) obviously holds, since

$$0 < \left(1 - e^{-(r+p)T}\right) < \left(1 - e^{-(r+2p)T}\right).$$

(b) If $A^{10} \leq B^{10} < 0$ we show numerically that (23) holds. In the numerical simulations we considered without loss of generality (see Remark 1) values $r = 0.05$ and $R = 1$. Using a grid 0.001×0.001 on the set of all positive (p, c), such that $p > 2c$ and $\frac{r+2p}{r+p}B^{10} < A^{10} \leq B^{10} < 0$, we plotted points where profit of the venture capitalist under contract $\mathcal{C}_6$ exceeds his profit under contract $\mathcal{C}_4$. The simulations show that this is the case everywhere in the defined domain. Figure 5 illustrates the case for $r = 0.05, p = 0.5$, where $\Delta := A^{10}\left(1 - e^{-(r+p)T}\right) - B^{10}\left(1 - e^{-(r+2p)T}\right)$.

3. According to Lemma 1, condition $A^{10}(r+p) < B^{10}(r+2p)$ implies that contract $\mathcal{C}_6$ is not feasible. Moreover, condition $(2c + E_0^{11}) < p(R + V_0^{11}) - 2c$ implies that $\mathcal{C}_5$ is feasible. Therefore, we choose the optimal contract between $\mathcal{C}_5$ and $\mathcal{C}_4$. Using numerical simulations, we have verified that in domain R_4, given that the feasibility conditions are satisfied for contract $\mathcal{C}_5$, the venture capitalist prefers to finance the leader alone (contract $\mathcal{C}_4$ is better than contract $\mathcal{C}_5$). Again, the numerical simulations were performed for $r = 0.05$ and $R = 1$, using a grid of 0.001×0.001 for parameters (p, c).

4. If $(2c + E_0^{11}) > p(R + V_0^{11}) - 2c$, the only feasible (hence, optimal) contract is $\mathcal{C}_4$.

□

B Appendix: Tables and figures

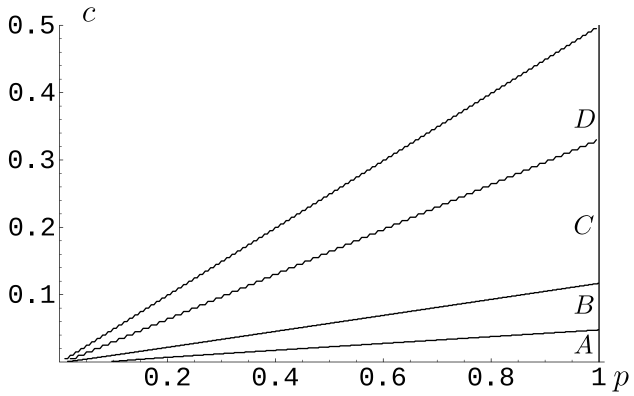

Figure 3: Feasibility of contracts in regime (1/0)

Notes to Figure 3:

1. Contract $\mathcal{C}_4$ is feasible in domains A,B,C and D;
2. Contract $\mathcal{C}_5$ is feasible in domains A, B and C;
3. Contract $\mathcal{C}_6$ is feasible in domain B;
4. Contract $\mathcal{C}_7$ is feasible in domain A.

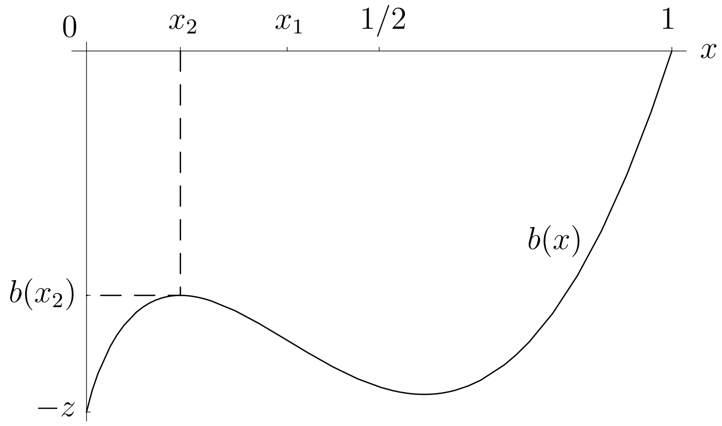

Figure 4: Shape of function $b(x)$

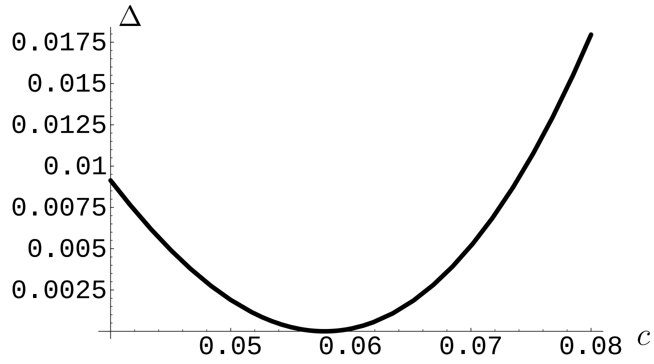

Figure 5: Regime (1/0): Illustration for Case 2; $p = 0.5$, $r = 0.05$.

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
Stoping rule	R_1	R_2	R_2	R_3
Share of entrep.	$s^{11}_{t,1} = \frac{c}{p} + E^{11}_t$	$s^{11}_{t,2} = \frac{c}{p} + E^{11}_t$	$s^{11}_{t,3} = \frac{c}{p} + E^{11}_t$	$s^1_t = \frac{c}{p} + E^1_t$
Value fnct. of entrep.	$E^{11}_{t,1} = \frac{c}{r+p} \cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$E^{11}_{t,2} = \left(\frac{1}{2}E^1_0 - \frac{c}{r+p}\right) \cdot$ $\cdot e^{-(r+p)(T-t)} + \frac{c}{r+p}$	$E^{11}_{t,3} = \frac{c}{r+p}$	$E^1_t = \frac{c}{r}\left(1 - e^{-r(T-t)}\right)$
Value of the venture	$V^{11}_{t,1} = \frac{2(Rp-c)}{r+2p} \cdot \left(1 - e^{-(r+2p)(T-t)}\right)$	$V^{11}_{t,2} = \left(V^1_0 - \frac{2(Rp-c)}{r+2p}\right) \cdot$ $\cdot e^{-(r+2p)(T-t)} + \frac{2(Rp-c)}{r+2p}$	$V^{11}_{t,3} = \frac{2(Rp-c)}{r+2p}$	$V^1_t = \frac{Rp-c}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$
Optimal time	$T^{11}_1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$	$T^{11}_2 = -\frac{1}{p} \ln \frac{r+p}{r+2p} \frac{E^1_0 - \frac{2c}{r+p}}{V^1_0 - \frac{2(Rp-c)}{r+2p}}$	$T^{11}_3 \to \infty$	$T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$
Feasibility condit.		$0 > A^{11}(r+p) > B^{11}(r+2p)$	$A^{11} > 0$	

Table 1: Optimal contracts and corresponding expected values in regime (1/1)

	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
Stoping rule	R_3	R_1	R_2	R_2
Share of the L.	$s_t^L = s_t^1 = \frac{c}{p} + E_t^1$	$s_{t,5}^L = \frac{c}{p} + E_{t,5}^F - E_0^{11}$	$s_{t,6}^L = \frac{c}{p} + E_{t,6}^L$	$s_{t,7}^L = \frac{c}{p} + E_{0,7}^L$
Value fnct. of the L.	$E_t^1 = \frac{c}{r}\left(1 - e^{-r(T-t)}\right)$	$E_{t,5}^L = \frac{c+pE_0^{11}}{r+p} \cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$E_{t,6}^L = \left(E_0^1 - \frac{c+pE_0^{11}}{r+p}\right) \cdot e^{-(r+p)(T-t)} + \frac{c+pE_0^{11}}{r+p}$	$E_{t,7}^L = \frac{c+pE_0^{11}}{r+p}$
Share of the F.		$s_{t,5}^F = \frac{c}{p} + E_{t,5}^F - E_0^{11}$	$s_{t,6}^F = \frac{c}{p} + E_{t,6}^F - E_0^{11}$	$s_{t,7}^F = \frac{c}{p} + E_{t,7}^F - E_0^{11}$
Value fnct. of the F.		$E_{t,5}^F = \frac{c}{r+p} \cdot (1 - e^{-(r+p)(T-t)})$	$E_{t,6}^F = \frac{c}{r+p}\left(1 - e^{-(r+p)(T-t)}\right)$	$E_{t,7}^F = \frac{c}{r+p}$
Value of the venture	$V_t^1 = \frac{Rp-c}{r+p} \cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$V_{t,5}^{10} = \frac{p(R+V_0^{11})-2c}{r+2p} \cdot \left(1 - e^{-(r+2p)(T-t)}\right)$	$V_{t,6}^{10} = \left(V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}\right) \cdot e^{-(r+2p)(T-t)} + \frac{p(R+V_0^{11})-2c}{r+2p}$	$V_{t,7}^{10} = \frac{p(R+V_0^{11})-2c}{r+2p}$
Optimal time	$T^1 = -\frac{1}{p}\ln\frac{c}{Rp-c}$	$T_5^{10} = -\frac{1}{p}\ln\frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}$	$T_6^{10} = -\frac{1}{p}\ln\frac{r+p}{r+2p}\ \frac{E_0^1 - \frac{2c+pE_0^{11}}{r+p}}{V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}}$	$T_7^{10} \to \infty$
Feasibility cond.		$2c + E_0^{11} < p(R + V_0^{11}) - 2c$	$0 > A^{10}(r+p) > B^{10}(r+2p)$	$A^{10} > 0$

Table 2: Optimal contracts and corresponding expected values in regime (1/0)