On Compatibility in Two-Sided Markets*

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Abstract

This paper provides a formal theory of compatibility choice between subsequent generations of technology in two-sided markets. We classify the compatibility regimes that can occur in two-sided markets. We explore how the decision of the monopolist to make technologies compatible (i.e., to choose a particular compatibility regime) depends on the characteristics of the market (size of installed base and market growth rate) and features of the new technology. The driving force that determines the choice of a compatibility regime is shown to be the tradeoff between incentives of the new agents on one side of the market and the incentives of the installed base on the other side of the market. Using this result, we characterize the choice of compatibility for three market structures: mature market, emerging market and asymmetric market. We show that compatibility for, say, users is likely to be imposed if the installed base of sellers is relatively small, the installed base of users is relatively small and the growth rate of this installed base is moderate. Further, the monopolist is less likely to improve compatibility if the technological progress is revolutionary. Our predictions about the choice of compatibility regime are illustrated by examples of particular two-sided markets.

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1 Introduction

The problem of compatibility choice in the framework of markets with network externalities has received much attention in the literature. This is not surprising, since the compatibility of products in this environment affects the size of relevant network and hence the incentives of agents to buy a particular product. Any decision of a firm operating in such market, from R&D to the introduction of upgrades, crucially depends on the fact whether its product is compatible with those of a rival or/and with the previous generations of the same product. It is surprising however, that, investigating compatibility choice, the literature did not pay much attention to the fact that many of the markets which exhibit network externalities are two-sided markets.[1]

Indeed, examples of two-sided markets are numerous. First of all, they include many industries of classical economy: newspapers and TV-channels, commercial fairs, dating agencies and night clubs, shopping malls, etc. However, the most prominent examples are related to the New Economy in general and to software platforms in particular. Operating systems, video-game consoles, payment cards, smart phones and PDA's all share features of two-sided (or, more generally, multi-sided markets). In a recent book, Evans, Hagiu and Schmalensee (2006) describe multi-sided software platforms as *invisible engines* that "are in the process of transforming industries ranging from automobiles to home entertainment" (p. vii).

In this paper we investigate the choice of compatibility between two generations of platforms (old and new) in the framework of two-sided markets. We provide classification of the compatibility regimes which one can observe on two-sided markets and develop a theory which explains how the choice of a particular regime depends on the characteristics of the market (the size of the installed base and the market growth rate) and technological features of the new platform. We show that the driving force which determines the choice of a compatibility regime is the tradeoff between incentives of the new agents on one side of the market and the incentives of the installed base on the other side of the market.

This paper is motivated by two observations. First, compatibility of technologies on two-sided markets has several regimes. Obviously, platforms may be incompatible with each other. GameCube, a video game console of Nintendo, is incompatible with its predecessor, N 64. Further, platforms may be backward compatible for one side of the market. Sony PlayStation 3, for example, is backward compatible with Sony PlayStation 2, its predecessor: a user of the former can play any games designed for the latter. Finally, platforms may be fully compatible with each other, as is the case with Palm OS. Not only a user of Palm OS can run on it any program designed for the older version of this operation system, but also any program designed for the new version of operation system can be run on the older version.

The second observation is that the choice of compatibility not only differs across industries (as illustrated by examples above) but (for the same firm) across time periods. As an example, consider Nintendo, which, after producing generations of

[1] Rochet and Tirole (forthcoming) define two-sided markets as markets, where one or several platforms enable interaction between two distinct group of agents and the volume of transaction is affected by a price structure.

incompatible game consoles, made its new game console, Wii, backward compatible with its predecessor, GameCube.

To explain these observations and to provide a theory of compatibility choice in two-sided markets we consider a framework with two platforms owned and operated by a single firm (referred to as the *monopolist*). The platforms enable interaction between two groups of agents, labelled as *users* and *sellers*. One of the platforms represents an old generation of technology and the other platform represents a new generation of technology. The new platform is superior to the old one in the extent of network benefits (which we also call *per-interaction benefits*) that it confers to users and sellers. In addition it has some intrinsic benefits, that are independent on the size of network, and reflect fashion or alternative uses of the platform. The size of network benefits and stand-alone benefits determine the extent of technological progress.

The old platform has an installed base: some users and sellers are already subscribed to this platform and can use it to interact with each other. In addition, there is a number of new users and sellers entering the market (their measure represents market growth rate). These new agents cannot subscribe to the old platform. To interact with agents on the other side of the market they have to, therefore, subscribe to the new platform. This indeed reflects the situation on many markets of interest, where the old generation of the platform (for example, an outdated operation system or an old generation of a game console) is no longer available (unless in a secondary market). The users and sellers are assumed to be heterogeneous with respect to net costs which they incur when adopting a new platform.

The price-discriminating monopolist earns profit by selling the new platform to the installed base and to the new agents and charging them a subscription fee. In addition the monopolist is free to choose among four compatibility regimes: making the new platform incompatible with the old one, fully compatible, or only backward compatible for agents on one side of the market. In the absence of any form of compatibility, only agents subscribed to the same generation of platform can interact. By imposing compatibility, the monopolist enables an interaction between users and sellers subscribed to different generations of platform.

Finally, deciding on compatibility, the monopolist can also determine the quality of interaction between agents, subscribed to the new platform and the agents on the other side of the market, subscribed to the old platform. The minimal quality which the monopolist can choose is zero, which corresponds to the situation where the new platform and the old platform are incompatible. It is assumed that the quality of interaction between agents subscribed to different platforms can never exceed the quality of interaction between agents subscribed to the old platform. In other words, the people who play a game designed for PlayStation 2 on PlayStation 3 can only enjoy the graphic and sound to the extent they would enjoy it using PlayStation 2. Any quality of interaction between zero and the maximal value corresponds to *partial compatibility*, because it only confers a part of maximal network benefits to agents on both sides od the market.

Our first crucial result is that the monopolist will never choose partial compatibility. He either will make technologies incompatible for one side on the market or will make them compatible to the extent that agents can enjoy the maximal network benefits. This result is new to the literature on network externalities, which up to now *assumed* that the compatibility is a yes/no decision (Katz and Shapiro 1985, Katz and Shapiro 1986, Farrell and Saloner 1986, Katz and Shapiro 1992, Doganoglu and Wright 2006),[2] although there always was an unease about this assumption (see, for example, Choi 1994). Our result provides a justification for this assumption and allows to concentrate our analysis on four extreme compatibility regimes (incompatible platforms, fully compatible platforms and two types of backward compatibility).

Analyzing the choice of the compatibility regime we identify three effects which drive the results. First, there is a *direct effect* of compatibility, which is positive for the new agents on one side of the market and is negative for the old agents on the other side of the market. For illustration consider a case, where monopolist makes platforms backward compatible for users. This improves incentives of new users to buy the new platform. Indeed, now, using this platform, they can access the installed base of sellers. On the other hand, the sellers, who belong to the installed base have less incentives to buy the new platform. Indeed, now, using their old platform they can interact with all users, subscribed to the new platform. Second, there is a *price effect*. As the monopolist improves compatibility of platforms for users he is also able to charge higher prices, which has a negative effect on users' demand.

The third effect is the negative *feedback effect* of compatibility. Decrease in the demand of old sellers leads to the decrease in the demand of new users and to the decrease in the demand of old users. The negative feedback effect becomes more important if the technological progress is revolutionary, while the direct positive effect less so. Indeed, if the new platform is very advanced, then the new users have large incentives to buy it even if it does not allow them to access the installed base of sellers. The compatibility therefore will bring only moderate improvement in their demand.

The tradeoff between *direct*, *price* and *feedback* effects determines which type of compatibility will be chosen on the market. To provide trackable analysis of compatibility choice, in the second part of the paper we concentrate on several market structures which are characterized by extreme values of one or several parameters and are observed in reality. These are mature market (the market growth rate is small), emerging market (the installed base is small) and the asymmetric market (the installed base exists only on one side of the market).

We characterize the optimal choice of compatibility for these chosen market structures. As follows from our analysis, the monopolist is more likely to make platforms compatible if the technological progress is moderate. Further, the compatibility for, say, users is likely to be imposed if the installed base of sellers is relatively small, the installed base of users is relatively small and the growth rate of their installed base is moderate.

Although our model is static we are able to provide some intuition about dynamics of compatibility choice as the market develops from emerging to mature or as

[2]One exception from this rule is Farrell and Saloner (1992), who assume that compatibility is provided through the use of converter, which can be imperfect. However, the quality of converter in their model is exogenously determined and is not chosen by firms, who provide converter.

the monopolist, who treated his market as one-sided business, embraces a two-sided model. We illustrate our predictions with examples from video game console market and market for personal digital assistants.

The set-up of our model shares common features with the literature on two-sided markets (Rochet and Tirole 2002, Caillaud and Jullien 2003, Armstrong forthcoming, Armstrong and Wright 2004, Rochet and Tirole forthcoming). Our results, however, are novel for this literature, which up to now did not devote much attention to the issue of compatibility. The exception is Doganoglu and Wright (2006), who investigate the incentives of competing firms to make their platforms compatible given that consumers of their products may (or may not) multihome, i.e. subscribe to both platforms. The authors mainly investigate markets with simple network externalities (i.e. there is only one group of agents). They, however, also discuss implications of their model to two-sided markets. The focus of this model is very different from ours. First, the incentives to make platforms compatible stem from competition. Second, the ability of consumers to multihome in their model is the driving force of the result, while in our model this is the tradeoff between incentives of old and new agents. Finally, the authors do not distinguish between different compatibility regimes and view compatibility as a yes/no decision (full compatibility/incompatibility).

The literature on compatibility in the presence of simple network externalities may be divided into two groups. The first group of papers investigate compatibility of technologies on perfectly competitive or oligopolistic market. The incentives of firms to make technologies compatible stem mostly from competition. Katz and Shapiro (1986) show that in a dynamic framework the competing firms have incentives to achieve compatibility of the products in order to soften the price competition on the early stage of the industry development. Kristiansen (1998) shows that compatibility may also be used to reduce the R&D competition at the stage of product introduction. Katz and Shapiro (1992) study a dynamic model, where consumers entering at each date choose between buying a incumbent technology or to wait until the entrant introduces more advanced technology. The authors show that, depending on the size of the installed base, market growth rate, and consumers' beliefs, either entrant or incumbent (but seldom both of them) would prefer to make both technologies compatible.

Unlike this strand of literature, we study the situation where both old and new platform (technology) are owned by a monopolist. We do this for two reasons. First, the structure on many industries involving multi-sided markets indeed is monopolistic (or close to monopolistic), for instance, PC operating systems with Microsoft, internet auctions with eBay, etc. Second, we want to analyze the incentives for achieving compatibility other than those which are related to competition. We show in the paper that incentives of the monopolist to make platforms compatible are determined by the extent to which he looses the demand on the behalf of the existing agents from one side of the market, which free-ride on the compatibility of platforms for agents on the other side of the market.

The second group of papers in the literature on network externalities is a literature on planned obsolescence. The paper which shares a number of similarities with our model in this literature is Choi (1994). This paper considers a decision of the monopolist in a two-period model. The monopolist sells a technology in the first period, forming an installed base, and a new generation of this technology in the second period. He has a choice between making the technologies compatible or incompatible with each other. Choi (1994) shows that the decision to introduce an incompatible technology crucially depends on the fact whether the monopolist intends to sell this technology to both installed base and new agents or only to new agents. In the former case the monopolist will make technologies incompatible, while in the latter case he will make them compatible. The first strategy (incompatible technology, sell to both groups of agents) is shown to be optimal if the new technology has sufficiently high stand-alone benefits and the first group of agents (installed base) is sufficiently large, compared to the number of new agents.

The intuition, underlying these results, that the tradeoff between the demand of old and new agents, is determinant for the compatibility decision, is similar to ours. Important difference, however, between this paper and Choi (1994) is that in our framework it is demand of the new agents on one side of the market and the demand of the old agents on the other side of the market which matters for compatibility choice. Further, in the framework of two-sided markets we are able to characterize the reacher set of compatibility regimes than Choi (1994). Finally, we also investigate how the choice of compatibility regime depends on the extent of network benefits, which the new technology confers to the agents on both sides of the market. Turns out that higher network benefits intensify the negative feedback effect while making the positive direct effect less important. This analysis allows to predict how the choice of compatibility changes with the technological progress and is missing (or, at least, is only implicit) in Choi (1994).

The remainder of the paper is organized as follows. In Section 2 we describe the setup of the model and provide a classification of compatibility regimes. Section 3 analyzes compatibility of platforms under a general demand specification. In Section 4 we introduce assumption of linear demand function and investigate three market structures: mature market, emerging market and asymmetric market. In Section 5 we illustrate our predictions about compatibility choice with two examples. Section 6 concludes. Appendix A contains proofs of all lemmas and propositions. Figures and tables are given in Appendix B.

2 Description of the model

There are two types of agents on the market: agents of type x and agents of type y. For simplicity we will often refer to the x-agents as *users* and to the y-agents as *sellers*. In line with the literature on two-sided markets, we assume that agents of each type derive utility from interacting with the agents of the other type, but not from interacting with the agents of their own type. The utility of each agent increases with the number of agents he can interact with.

In order to *interact with* (*connect to*) an agent of type j, an agent of type i (where $i, j \in \{x, y\}$, $i \neq j$) needs to be subscribed to a *platform*. There are two different platforms available: Platform 0 and Platform 1. Platform 0 represents the old (default) technology and Platform 1 represents the new technology.[3] Both platforms are operated by a single monopolistic firm, which also retains all profits generated by the platforms. The extent to which these platforms differ will become clear later.

We present our model in a general setting which allows to derive general results for two-sided markets. In the Introduction we have described numerous examples of such two-sided markets. Although, each specific example may involve some properties not captured by the model, the mechanisms described in this paper remain at work in most cases. As a typical example to illustrate the assumptions on the technology, we will use the market for video-game consoles. Agents of type x (users) are represented by the players of video-games and agents of type y (sellers) are represented by software developers. A platform in such a market is a video-game console, which enables users to play games, developed by software developers. Old technology then corresponds to an old generation of the console (e.g., Sony PlayStation 2) and the new technology corresponds to a new generation of the console (e.g., Sony PlayStation 3).

We assume that there are non-negative measures b^x and b^y of users and sellers respectively, who are already subscribed to the old platform (Platform 0). We will refer to these agents as *existing members*, *old agents*, or *installed base*. In addition, there are measures c^x and c^y of *new agents* (of types x and type y respectively) who are not subscribed to any platform. We assume that these agents cannot subscribe to the old technology. Their only way to connect to the agents of the other type is to subscribe to the new technology. This assumption reflects the situation where the old technology is discontinued (no longer available) and has been replaced by the new technology. For example, in case of video-game consoles, after introducing a new console, the old one cannot be purchased (unless in a secondary market).

The existing members, already subscribed to Platform 0, may also subscribe in addition to Platform 1. In this case, the agents retain also the old technology[4] and may use either the new or the old technology to interact with the agents of the other type. This assumption is justified in the situation where parallel use of two platforms generates no or only negligible additional costs. Note that if there is no possibility of resale and the use of the old platform does not involve any additional costs, agents who subscribed to the new platform have no incentives to stop using the old one.

Subscription to the new technology may be beneficial due to two reasons. First, it may involve some technological advantages, like technological parameters (better graphics, sound, etc.) or alternative uses (as a DVD player), that increase the utility from interaction. Second, it may enable interaction with agents, not subscribed to the default technology.

If two agents interact using the old technology, their benefit from this interaction is normalized to 1. This implies that users and sellers that are subscribed to the old technology, are guaranteed to receive the benefit of b^y (respectively b^x) by interacting with the agents from the opposite group, who are also subscribed to this technology. We will further assume that when interaction is realized through the new platform

[3] We will often use the words *platform* and *technology* interchangeably.

[4] This is often called *multihoming*.

(Platform 1), the benefit from this interaction is scaled up by a constant factor $s \geq 1$. Hence, new technology is beneficial for both users and sellers, because it allows to extract a higher utility from the same number of interactions. Finally, if the two technologies are compatible, also agents using different technologies can interact. We assume that in this case the benefits of interaction are determined by the lowest technology which enables the interaction. For example, when old games can be played in the new console, there is usually no additional benefit compared to the old console.[5]

The profit-maximizing monopolist, who owns and operates both platforms, makes profit by charging per-subscription prices for Platform 1. The whole situation is then modelled as a three-stage game: in the first stage the monopolist chooses the compatibility regime, in the second stage he chooses the prices, and in the third stage the agents simultaneously decide whether to subscribe to Platform 1. As a solution concept we use subgame-perfect Nash equilibrium.[6] We denote A_0^i the price charged to installed base, A_1^i the price charged to new agents ($i \in \{x, y\}$). Observe that we allow for price discrimination, i.e., the monopolist can charge different prices to old and new agents of the same type. This can be achieved by selling the new platform in form of an *update* to the old platform with a different price than the stand-alone platform. The assumption of price discrimination is in line with existing literature on network externalities (Ellison and Fudenberg 2000, Choi 1994). The most prominent example are the rebates for the users of operating systems for updates.

For simplicity, we will assume that there is no cost of operating a platform. The monopolist cannot charge a price for Platform 0 (it is not any more available for sale) and cannot make any profit on those agents which use the old technology. Let $m^i \in [0, 1]$ denote the share of the existing members that subscribe to Platform 1 and let $n^i \in [0, 1]$ denote the share of the new agents that subscribe to Platform 1. Then $b^i m^i$ and $c^i n^i$ are their demands for Platform 1. Since the monopolist has no costs and charges per subscription prices, his profit is

$$\Pi = A_0^x b^x m^x + A_0^y b^y m^y + A_1^x c^x n^x + A_1^y c^y n^y. \tag{1}$$

The model we presented so far is a modification of the traditional model of two-sided markets (Rochet and Tirole 2004). Our main contribution is the analysis of monopolist's decision about compatibility between the old and the new technology. There are four possible *compatibility regimes*. The new technology may be *incompatible* (NC) with the old technology. It may be *backward compatible* with the old technology for x-agents (BCx); it may be backward compatible for y-agents (BCy) or it may be *fully compatible* with the old technology (FC).

Under no-compatibility, the new and old technologies cannot be interconnected. Backward compatibility for i-agents means that an i-agent, who is subscribed to the

[5]If we define something like quality of the platform, then this amounts to assuming that benefit from the interaction is determined by the minimal quality of platforms used. This is an assumption on the technology of interactions. Alternatively, one could consider the benefits from interactions to be determined by the maximum of qualities, some convex combination of qualities, or by own type's quality.

[6]Further, we will call it only *equilibrium*. We will also refer to the monopolist's choice in the second stage given the compatibility regime (i.e., equilibrium in the second stage) as *optimum* or *maximum*.

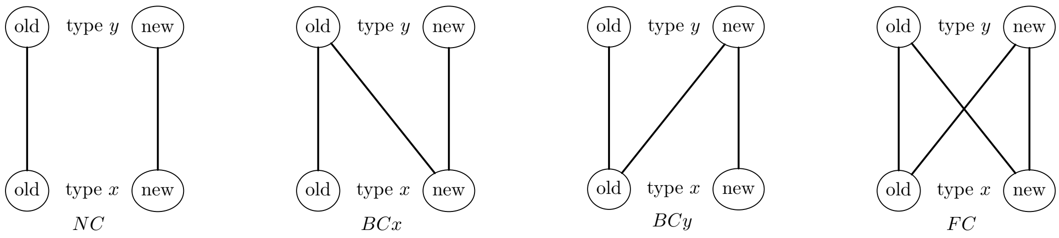


Figure 1: Interactions in various compatibility regimes

new technology may use it to interact with an j-agent subscribed to the old technology (see Figure 1). In the example of game consoles, backward compatibility for users means that games produced for the old console (PlayStation 2) can be played on the new console (PlayStation 3). In technical language, this form of compatibility is simply called "backward compatibility." A related notion of "forward compatibility" means that games written for the new console can be played using the old console.[7] In our setting of two-sided markets, forward compatibility is equivalent to backward compatibility for sellers — it simply means that sellers subscribed to the new technology can interact with users subscribed to the old technology. Finally, if the new technology is backward compatible for both sides of the market, we say that the technology is fully compatible. Example of full compatibility is the USB standard: USB 2.0 is fully compatible with USB 1.1.

All these compatibility regimes can be easily nested within one general framework. Towards this end let us assume that the benefit from interaction of a new x-agent with an old y-agent is γ^x and the benefit from interaction of a new y-agent with an old x-agent is γ^y.[8] Thus, γ^x and γ^y can interpreted as *degrees of backward compatibility* for x-agents and y-agents. The value $\gamma^x = 0$ means that the benefit from interaction (between a new x-agent and an old y-agent) is 0, i.e., the new platform is not backward compatible for x-agents. On the other hand, the value $\gamma^x = 1$ means this benefit is 1, i.e., the new platform is backward compatible for x-agents. The regime NC then corresponds to the case $\gamma^x = \gamma^y = 0$, regime BCx to $\gamma^x = 1$ and $\gamma^y = 0$, regime BCy to $\gamma^x = 0$ and $\gamma^y = 1$, and regime FC to $\gamma^x = \gamma^y = 1$. We will refer to the case when γ^x or γ^y belong to $(0, 1)$ as *partial compatibility.* As will be shown below (Proposition 1), partial compatibility never chosen by the monopolist, even if he is free to choose any $\gamma^x, \gamma^y \in [0, 1]$. This provides justification for using only the polar cases NC, BCx, BCy, and FC.

If an agent of type $i \in \{x, y\}$ does not subscribe to the new platform, his utility is simply equal to his benefit from interactions. We denote U_0^i the utility of the old agent not subscribed to the new platform; the utility of the new agents not subscribed to any platform is normalized to zero. If the agent subscribes to the new platform, his utility depends positively on the per-interaction benefits, negatively on the subscription price

[7]For more details see `en.wikipedia.org/wiki/Backward_compatibility` and `en.wikipedia.org/wiki/Forward_compatibility`.

[8]Alternatively this can mean that a new x-agent is able to connect only to a share of γ^x of the old y-agents and *vice versa*.

A_k^i (where $k \in \{0,1\}$) and on intrinsic benefits or costs of acquiring the new platform.[9] We will summarize those by θ^i, $i \in \{x,y\}$ that represents the *net costs* of acquiring the new platform. The utility function is assumed to be additive and, thus, equal to

$$(\text{benefit from interactions}) - \theta^i - A_k^i.$$

We denote U_1^i and V_1^i the utility of old and new agents respectively who are subscribed to the new platform.

Benefits which an agent derives from interactions depend on the degree of compatibility. For illustration, consider an old agent of type x. If he joins Platform 1, he can interact with $b^y m^y + c^y n^y$ agents using Platform 1 (with per-interaction benefit s) and with the remaining $b^y(1-m^y)$ agents using Platform 0 (with per-interaction benefit 1). Thus, his benefit from interactions is $s(b^y m^y + c^y n^y) + b^y(1-m^y)$. Here, degree of compatibility plays no role. On the other hand, if he does not join Platform 1, he can interact with b^y old agents using Platform 0 (with per-interaction benefit 1) and also with $c^y n^y$ new y-agents (with per-interaction benefit γ^y). In that case, his benefit from interactions is $b^y + \gamma^y c^y n^y$. Formally, the agent's utilities are

$$U_1^x = s(b^y m^y + c^y n^y) + b^y(1-m^y) - \theta^x - A_0^x, \qquad U_0^x = b^y + \gamma^y c^y n^y.$$

A new agent of type x can stay out of the market in which case he has no access to the agents of type y and receives zero benefits from interactions. Alternatively, he can subscribe to the new platform. Platform 1 enables him to interact with $c^y n^y$ new agents (with per-interaction benefit s) and with additional $b^y m^y$ old agents (with per-interaction benefit γ^x). Formally,

$$V_1^x = \gamma^x b^y + (s-\gamma^x) b^y m^y + s c^y n^y - \theta^x - A_1^x.$$

The demand for Platform 1 is given by the number (measure) of existing members for which $U_1^i > U_0^i$ and the number (measure) of new agents for which $V_1^i > 0$. In particular,

$$U_1^x > U_0^x \iff (s-1) b^y m^y + (s-\gamma^y) c^y n^y - A_0^x > \theta^x, \quad (2)$$
$$V_1^x > 0 \iff \gamma^x b^y + (s-\gamma^x) b^y m^y + s c^y n^y - A_1^x > \theta^x. \quad (3)$$

All existing x-agents with θ^x satisfying the former inequality and all new x-agents with θ^x satisfying the latter inequality will subscribe to Platform 1.

Comparison across different compatibility regimes reveals the twofold effect which compatibility has on the incentives of agents. BCx as compared to NC (or in general increase in γ^x), for example, increases incentives of new users to subscribe to the Platform 1 by enabling them to access the larger population of agents on the other side of the market. On the other hand, BCx regime (or increase in γ^x) discourages existing sellers to buy the new technology. Indeed, in this regime they can interact with users using their old platform. This tradeoff between incentives of the new

[9] The intrinsic benefits may reflect alternative uses of the platform (Sony PlayStation can be used as a DVD player) or fashion.The cost may represent switching costs.

agents on one side of the market and old agents on the other side of the market will be determinant for the choice of the compatibility regime.

The agents are assumed to be heterogenous with respect to the net costs θ^i; let $F(\theta^i)$ be its cumulative distribution function. We assume that the distribution of agents' net costs has a finite support $[\underline{\theta}, \bar{\theta}]$. In addition, function F is increasing and twice continuously differentiable on $[\underline{\theta}, \bar{\theta}]$, and the following assumption hold. Note that under the introduced specification, the net costs the old and new agents have the same distribution of net benefits, reflecting the fact that the new agents are a "copy" of the old agents. For simplicity, we also assume the same distribution of costs on both sides of the market. This setup allows to analyze monopolist's decision based on the markets sizes b^i and c^i. It is straightforward to modify the model in order to allow for different distributions. All results remain valid, however, at the expense of simplicity of some conditions.[10]

Assumption 1. $F'(\bar{\theta}) = 0$ and $\lim_{\theta \to \underline{\theta}^+} \dfrac{F(\theta)}{F'(\theta)} < -\underline{\theta}$.

As will be shown later, Assumption 1 guarantees existence of interior solution to the monopolist's maximization problem.[11] The first inequality implies that there is no kink at point $\bar{\theta}$ and hence we may use first-order conditions to find the maximal profit.[12] The second inequality requires $\underline{\theta} < 0$, which means that there is some group of agents who derive (positive) net benefits from the new technology. These agents then ensure that all demands are positive in equilibrium. Note that the second inequality holds whenever $\underline{\theta} < 0$ and $\lim_{\theta \to \underline{\theta}^+} F'(\theta) > 0$.

It follows from (2) and (3) that the demands of old and new x-agents and by a symmetric argument also of old and new y-agents are given by

$$m^x = F\big((s-1)b^y m^y + (s-\gamma^y)c^y n^y - A_0^x\big), \tag{4}$$
$$m^y = F\big((s-1)b^x m^x + (s-\gamma^x)c^x n^x - A_0^y\big), \tag{5}$$
$$n^x = F\big(\gamma^x b^y + (s-\gamma^x)b^y m^y + c^y n^y - A_1^x\big). \tag{6}$$
$$n^y = F\big(\gamma^y b^x + (s-\gamma^x)b^x m^x + c^x n^x - A_1^y\big). \tag{7}$$

Note, that if there is no entry to the market (i.e., $c^x = c^y = 0$), then all compatibility regimes result in the same demand for Platform 1. Therefore, in the absence of new agents, the monopolist is indifferent between the four compatibility regimes.[13]

[10] To analyze the model with different distributions for each type of agents, fix the markets sizes and analyze the decision with respect to demand elasticities.

[11] This holds for all values of other parameters. In Section 4 we consider F linear on $[\underline{\theta}, \bar{\theta}]$ that satisfies the second inequality, but violates the first inequality. Thus, some corner solutions arise.

[12] Note that this assumption is by no means restrictive. Indeed, any function can be "smoothed" in a small neighborhood of $\bar{\theta}$ so that the first and second derivatives become continuous.

[13] If imposing compatibility involves some small fixed costs, then in the absence of entry, the monopolist has no incentives to make the platforms compatible.

3 General demand function

In this section we analyze the general model introduced above. For simplicity of notations we denote $G : [0,1] \to [\underline{\theta}, \bar{\theta}]$ the inverse function to F. By assumptions, G is increasing and twice continuously differentiable. Our assumption on the distribution of θ^i implies that $G(m^i)$ and $G(n^i)$ represent the characteristic θ^i of the indifferent existing member and new agent respectively. Note that $G(0) = \underline{\theta}$ and $G(1) = \bar{\theta}$. In order to simplify the notation, it will be convenient to use function $H : [0,1] \to \mathbb{R}$ such that

$$H(z) = \tfrac{1}{2} z G(z) \qquad \text{for all } z \in [0,1]. \tag{8}$$

With this notation, we obtain inverse demands (prices as functions of demands):

$$\begin{aligned}
A_0^x &= -G(m^x) + (s-1) b^y m^y + (s - \gamma^y) c^y n^y, \\
A_0^y &= -G(m^y) + (s-1) b^x m^x + (s - \gamma^x) c^x n^x, \\
A_1^x &= -G(n^x) + \gamma^x b^y + (s - \gamma^x) b^y m^y + c^y n^y, \\
A_1^y &= -G(n^y) + \gamma^y b^x + (s - \gamma^x) b^x m^x + c^x n^x.
\end{aligned}$$

and the monopolist's profit becomes

$$\begin{aligned}
\Pi = -[b^x m^x G(m^x) + b^y m^y G(m^y) + c^x n^x G(c^x) + c^y n^y G(c^y)] + \\
+ 2(s-1) b^x b^y m^x m^y + 2(s - \gamma^x) b^y c^x m^y n^x + 2(s - \gamma^y) b^x c^y m^x n^y + 2 s c^x c^y n^x n^y + \\
+ \gamma^x b^y c^x n^x + \gamma^y b^x c^y n^y.
\end{aligned} \tag{9}$$

This is to be maximized with respect to $m^x, m^y, n^x, n^y \in [0,1]$. We may immediately observe that the "coefficient" at γ^x is $b^y c^x n^x (1 - 2m^y)$. As will become clear below, the comparison of m^y to $\frac{1}{2}$ will be important for monopolist's decision whether to impose backward compatibility for x-agents.

The following lemmas provide sufficient conditions for existence and uniqueness of the maximum.

Assumption 2. Function $H''(z)$ is bounded from below on $[0,1]$ and its minimum $\Delta = \min_{z \in [0,1]} H''(z)$ satisfies the following conditions:

(i) $\Delta > 0$;

(ii) $\Delta^2 > (s-1)^2 b^x b^y + s^2 b^y c^x$; and

(iii) $\Delta^4 + s^2 b^x b^y c^x c^y > [(s-1)^2 b^x b^y + s^2 b^y c^x + s^2 b^x c^y + s^2 c^x c^y] \Delta^2$.

Lemma 1. *If Assumption 2 is satisfied, then monopolist's profit* (9) *is concave in* m^x, m^y, n^x, n^y *for all* $\gamma^x, \gamma^y \in [0,1]$.

Lemma 2. *If Assumptions 1 and 2 are satisfied, then for all* $\gamma^x, \gamma^y \in [0,1]$*, the monopolist's profit* (9) *has a unique maximum. This maximum is achieved for* m^x*,* m^y*,* n^x*, and* n^y *from* $(0,1)$ *that satisfy the first-order conditions*

$$\begin{aligned}
H'(m^x) &= (s-1) b^y m^y + (s - \gamma^y) c^y n^y, \\
H'(m^y) &= (s-1) b^x m^x + (s - \gamma^x) c^x n^x, \\
H'(n^x) &= \tfrac{1}{2} \gamma^x b^y + (s - \gamma^x) b^y m^y + s c^y n^y, \\
H'(n^y) &= \tfrac{1}{2} \gamma^y b^x + (s - \gamma^x) b^x m^x + s c^x n^x.
\end{aligned}$$

In what follows in this section we will assume that both Assumptions 1 and 2 are satisfied. We will focus on the analysis of the optimum. A natural question arises, what would be the optimal choice of γ^x and γ^y if the monopolist could choose any values from $[0, 1]$. The answer is surprisingly that the monopolist would never choose $\gamma^x \in (0, 1)$ and is formulated in Proposition 1. Furthermore, Proposition 2 provides sufficient conditions for comparisons of compatibility regimes. We will use the relation "$\prec$" to denote the comparison of monopolist's optimal profit across compatibility regimes.

Proposition 1. *Partial compatibility is never optimal.*

Proposition 2. *The following statements hold for comparison of optimums in compatibility regimes:*

(i) *If $m^x \leq \frac{1}{2}$ in NC (resp. BCx) regime, then $NC \prec BCy$ (resp. $BCx \prec FC$).*

(ii) *If $m^x \geq \frac{1}{2}$ in BCy (resp. FC) regime, then $NC \succ BCy$ (resp. $BCx \succ FC$).*

(iii) *If $m^y \leq \frac{1}{2}$ in NC (resp. BCy) regime, then $NC \prec BCx$ (resp. $BCy \prec FC$).*

(iv) *If $m^y \geq \frac{1}{2}$ in BCx (resp. FC) regime, then $NC \succ BCx$ (resp. $BCy \succ FC$).*

Proposition 1 provides a justification for analyzing only the polar cases: NC, BCx, BCy, and FC. The intuition behind Proposition 1 is based on a stronger statement. Namely, the monopolist has incentives to reduce the degree of backward compatibility for x-agents, if sufficiently many old y-agents subscribe to the new platform. Sufficiently many here means that the median old y-agent, i.e., agent with costs θ that satisfy $F(\theta) = \frac{1}{2}$, will in optimum subscribe to the new platform. This reminds on the *Median Voter Theorem* in the sense that the median agent is determinant for the compatibility choice. On the other hand, if the median old y-agent does not subscribe to the new platform, the monopolist has incentives to increase the degree of compatibility for x-agents. Thus, the only candidate for optimum that remains is when the median old y-agent is exactly indifferent between subscribing and not subscribing to the new platform. However, in that case the reduction in γ^x has a positive effect on old y-agents' incentives to subscribe to the new platform and the monopolist is again willing to reduce the degree of compatibility. Therefore, the monopolist never chooses a partial degree of compatibility for x-agents.

The remaining sufficient conditions in Proposition 2 follow the same intuition. Note that the sufficient conditions are formulated in terms of the median old y-agent. This is intuitive, since the measure of old y-agents who join the new platform determines the incentives of new x-agents. Analogous statements can be also made for backward compatibility for y-agents.

Unfortunately, under with the general form of distribution of agents' costs, it is not possible to provide a complete characterization of the monopolist's compatibility choice in terms of the primitives of the model (i.e., market sizes b^x, b^y, c^x, c^y, quality of connection s and distribution of agents' costs represented by function F). In the following section we consider a uniform distribution of agents' costs (i.e., linear F) that allows for explicit solutions. However, even in the general case, we are able to derive some comparative statics results. In particular, we are interested in comparative statics with respect to γ^x (with symmetric results for γ^y). It provides an important intuition on the effects which drive a choice of compatibility regimes. We can easily evaluate that all derivatives $\mathrm{d}m^x/\mathrm{d}\gamma^x$, $\mathrm{d}m^y/\mathrm{d}\gamma^x$, $\mathrm{d}n^x/\mathrm{d}\gamma^x$, $\mathrm{d}n^y/\mathrm{d}\gamma^x$ can be written in the form

$$b^y(1-2m^y)\beta_1 + 2c^x n^x \beta_2, \qquad \text{where } \beta_1 > 0 \text{ and } \beta_2 < 0$$

(coefficients β_1 and β_2 differ across variables, but their signs are always as indicated; see the proof of Proposition 3 for technical details). Thus, the effect of an increase in γ^x can be decomposed in two effects. The first effect stems from the change in incentives of new x-agents and is negative or positive depending on whether the median old y-agent subscribes to the new platform or not. The second stems from the change in incentives of old y-agents and is always negative.

Decomposing the first effect into three part we can understand the source of its ambiguity. The increase in γ^x has a *direct effect* which is positive for new x-agents and is negative for old y-agents. The second effect is the *negative feedback effect*: reduction in the demand of old y-agents triggers the reduction in demand of new x-agents.[14] Finally, there is a *price effect*: if the monopolist improves compatibility of platforms for x-agents he is also able to charge higher prices for the access to this platform. The higher price has negative on the incentives of new x-agents to buy a platform. Intuitively, if m^y is large the negative effects (feedback effect and price effect) should overweight the positive effect. Indeed, in this case sufficiently many old y-agents purchase the new platform to make it attractive for the new x-agents even in the absence of compatibility. Hence, if compatibility is improved, the direct positive effect will be negligible and will be dominated by negative effects. Since the direct effect of compatibility on the incentives of the old y-agents is always negative, the total effect will be negative as well. This intuition is formalized in the following proposition. In a similar way we may derive comparative statics result with respect to γ^y.

Proposition 3. *For any $\gamma^y \in [0,1]$, all demands (m^x, m^y, n^x, and n^y) and also the monopolist's profit are decreasing in γ^x whenever $m^y \geq \frac{1}{2}$ in optimum.*

4 Compatibility choice with linear demand functions

To investigate the choice of compatibility in more details we will make here an additional assumption that θ^i (where $i \in \{x,y\}$) is uniformly distributed on the interval $[-B,\, 1-B]$, where $0 < B < 1$. This means, that there are some agents who derive net benefit and some agents who derive net costs from the new platform. The value of B then corresponds to the maximal net benefit (derived by agents with $\theta = -B$).

[14] It becomes clear later that demand of the old x-agents is also subject to negative feedback effect.

In the notation of Section 2 the corresponding distribution function is $F(\theta) = \theta + B$ for $\theta \in [-B,\, 1-B]$. The demands are then linear in prices and are given by (4)–(7).

To ensure that the demand functions are decreasing in prices, we will impose two conditions on parameters.

Assumption 3. $b^x \leq 1, b^y \leq 1, c^x \leq 1, c^y \leq 1$.

Assumption 4. $1 - b^x b^y (s-1)^2 - s^2(b^y c^x + c^x c^y + b^x c^y - b^x b^y c^x c^y) > 0$.

These conditions are sufficient conditions on parameters and (for linear demand function) imply conditions in Lemma 1. Indeed, for linear demand function $\Delta = 1$. Hence, condition (iii) in Lemma 1 becomes identical to Assumption 4. Further, if Assumptions 4 and 3 are satisfied, then condition (ii) of Lemma 1 is satisfied.

Since in equilibrium each demand can be either interior or corner, we have multiple candidates for equilibrium allocation. However, it is possible to show, that the monopolist never chooses prices, such that $m^i = 0$ or $n^i = 0$ or both. Moreover, the interior solution maximizes the principal's profit, whenever feasible.

Lemma 3. *Consider the maximization of monopolist's profit with respect to m^x, when keeping $m^y, n^x, n^y \in [0,1]$ fixed. Let $m^{x*} \in \mathbb{R}$ solve the first-order condition $\partial \Pi^r / \partial m^x = 0$, where $r \in \{NC, BCx, BCy, FC\}$. Then, the following statements hold:*

(i) $m^{x*} > 0$.

(ii) *If $m^{x*} \in [0,1]$, then $m^x = m^{x*}$ maximizes monopolist's profit when keeping m^y, n^x, n^y fixed.*

(iii) *If $m^{x*} > 1$, then $m^x = 1$ maximizes monopolist's profit when keeping m^y, n^x, n^y fixed.*

Analogous statements holds for (partial) maximization with respect to any of the variables m^y, n^x, n^y.

Based on the Lemma 3, we can eliminate allocations where either $m^i = 0$ or $n^i = 0$ for any $i \in \{x, y\}$ from the set of equilibrium candidates. This leaves us sixteen allocations, which are candidates for equilibrium. These allocations are summarized in Table 5 in Appendix B. Notations are as follows: $E_{i_1 i_2 i_3 i_4}$ denotes a particular type of allocation. Indexes $i_1 \in \{0,1\}$, $i_2 \in \{0,1\}$, $i_3 \in \{0,1\}$ and $i_4 \in \{0,1\}$ indicate, whether in this allocation the demands m^x, m^y, n^x and n^y respectively are interior ($i_k = 0$) or corner ($i_k = 1$). For example, the allocation where $m^x \in (0,1)$, $m^y \in (0,1)$, $n^x = 1$, and $n^y \in (0,1)$ is denoted as E_{0010}.

The use of linear demand functions allows us to derive equilibrium prices, demands and the monopolist's profit in the closed form. However, with six parameters the presentation of results in general case (that is $b^i > 0$, $c^i > 0$, $s > 1$, and $0 < B < 1$) does not reveal the underlying intuition. Therefore, in what follows we discuss several specific market structures that one can observe in reality, and that are characterized by extreme values of one or several parameters.

4.1 Mature market

We define a mature market (or satiated market) as a market with low growth rate, i.e. we assume that c^x and c^y are close to zero. An example of such market is the market for Microsoft Windows. The operation system is installed on more than 90% of all computers, hence there are very few users who do not belong to the installed base of Microsoft and very few software developers who do not adapt their applications for Windows.[15]

Recall, that in our model at the market where $c^x = c^y = 0$ all compatibility regimes are equivalent in terms of monopolist's profit, agents' demands and prices for the new technology. Using this result we can compare different compatibility regimes at the market where the number of new agents is small, investigating the monopolist's profit in the neighborhood of $(c^x, c^y) = (0, 0)$. The comparison is summarized in the following proposition. Let us first define:

$$g_1(z) = \frac{1 - B[1 + z(s-1)]}{z(s-1)^2}.$$

Proposition 4. *Assume that c^x, c^y are sufficiently small (close to zero). Then the following implications hold:*

$$\begin{aligned}
b^x < \frac{1-B}{2(s-1)} \quad &\textit{or} \quad b^y < g_1(b^x) &\implies \quad &NC \prec BCx \quad \textit{and} \quad BCy \prec FC,\\
b^x > \frac{1-B}{2(s-1)} \quad &\textit{and} \ b^y > g_1(b^x) &\implies \quad &NC \succ BCx \quad \textit{and} \quad BCy \succ FC,\\
b^y < \frac{1-B}{2(s-1)} \quad &\textit{or} \quad b^x < g_1(b^y) &\implies \quad &NC \prec BCy \quad \textit{and} \quad BCx \prec FC,\\
b^y > \frac{1-B}{2(s-1)} \quad &\textit{and} \ b^x > g_1(b^y) &\implies \quad &NC \succ BCy \quad \textit{and} \quad BCx \succ FC.
\end{aligned}$$

Using the above proposition we can define indifference curves[16] $\mathcal{I}^x$ and $\mathcal{I}^y$ which represent the monopolist's indifference between providing and not providing backward compatibility for agents of x-type and y-type respectively:

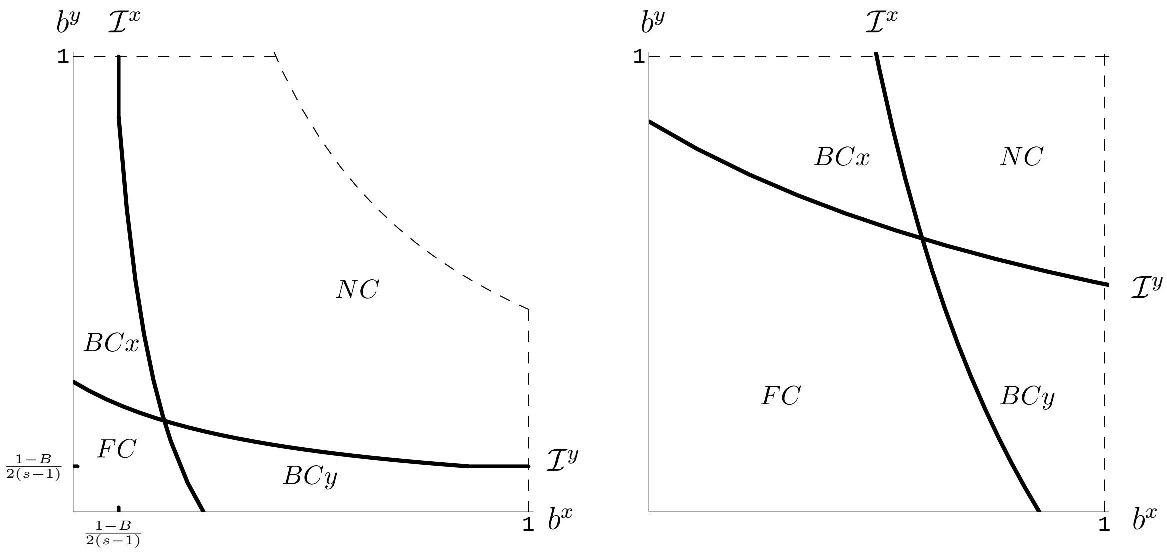
$$\mathcal{I}^y = \begin{cases} \frac{1-B}{2(s-1)}, & \text{if } b^y \leq \frac{1-B}{2(s-1)}, \\ g_1(b^y), & \text{if } b^y > \frac{1-B}{2(s-1)}. \end{cases} \tag{10}$$

$\mathcal{I}^x$ is analogically defined.[17] Notice, that $\mathcal{I}^y$ decreases in B, s and is non-increasing in b^y (analogical result holds for $\mathcal{I}^x$). The indifference curves and optimal choice of compatibility regime is illustrated on the Figure 2 (the dashed line shows the area where Assumption 3 and Assumption 4 are satisfied).

[15]See `en.wikipedia.org/wiki/Microsoft_Windows`

[16]With some abuse of notation we use the same notation for the indifference curve and for the function describing it.

[17]These indifference curves need to be interpreted properly. Consider, for example, curve $\mathcal{I}^y$. If (b^x, b^y) lies above (resp. below) the curve $\mathcal{I}^y$, then there exists $\delta > 0$ such that the monopolist prefers NC to BCy (resp. prefers BCy to NC) for all $c^x, c^y \in (0, \delta)$.



(a) $s = 2.5$, $B = 0.7$ **(b)** $s = 1.5$, $B = 0.7$

Figure 2: Optimal choice of compatibility regimes at the mature market

There are several observations to be made about the choice of compatibility regime. First, as we have already mentioned, backward compatibility for, say, y-agents improves incentives of new agents of this type to buy the new technology, while it discourages the old x-agents to buy the new technology (*direct effect*). Moreover, the decrease in demand on behalf of old x-agents leads to the decrease in demand of new y-agents and old y-agents (*negative feedback effect*). In addition, there is a negative *price effect*.

Therefore, whether the monopolist is willing to make technologies compatible for y-agents depends on whether the positive effect on behalf of new y-agents overweighs the negative effects. This tradeoff explains the fact that for each b^x there exist a cutoff value of b^y (defined by $\mathcal{I}^y$), such that for all b^y above this value the monopolist will make technologies not compatible for y-agents. Moreover, this cutoff value is non-increasing in b^x, since the larger is b^x the more important is the negative effect of backward compatibility for the monopolist's profit. On the other hand, if b^y is sufficiently small (when $b^y < \frac{1-B}{2(s-1)}$), the feedback effect on old y-agents becomes negligible and the monopolist is willing to make technologies backward compatible for any value of b^y. In particular, for $s = 1$ the negative feedback effect vanishes. Indeed, if $s = 1$ the demand of old y-agents does not depend on demand of old x-agents (one can readily see this from the definition of demand functions). In this case FC regime will be always chosen in equilibrium.

Another observation is that the larger is the technological progress, the less willing is the monopolist to make technologies compatible. This follows directly from the fact, that $\mathcal{I}^x$ and $\mathcal{I}^y$ decrease in B (value of stand-alone benefits) and s (per-interaction benefits). The underlying intuition is that the better is the new technology, the more incentives have the new agents to purchase it even in the absence of compatibility. On the other hand, the reduction in demand on behalf of old agents becomes more important for the principal's profit as s or/and B increase. This observation is formalized in the following corollary.

Corollary 1. *Consider some* $b^x, b^y > 0$*. Let* $\bar{s} > 1$ *and* $\bar{B} \in (0,1)$ *satisfy Assumption 4 (where* $c^x \to 0$ *and* $c^y \to 0$*). If the monopolist makes technologies compatible for* $s = \bar{s}$ *and* $B = \bar{B}$*, then he will make technologies compatible for all* $s \leq \bar{s}$ *and* $B \leq \bar{B}$*.*

Finally, as follows from the Proposition 4, the decision of the monopolist whether to make technologies compatible for agents of type x does not depend on the fact, whether they are already compatible for the agents of type y. In other words, if the monopolist decides to switch from NC regime to BCx regime, he would also switch from BCy regime to FC regime (the symmetric argument holds the other side of the market). This result amounts to saying that the change in demand of new agents of type y is negligible and does not play a role for the decision of the monopolist to introduce BCx regime. Indeed, for $c^y \to 0$ this effect is insignificant and is dominated by the direct effect of BCx regime (negative effect on the incentives of old agents of type y and positive effect on the incentives of new agents of type x) and indirect feedback effect.

4.2 Emerging market

We define an emerging market as the market with very small installed base ($b^x \to 0$, $b^y \to 0$) and potentially high growth rate ($c^x > 0, c^y > 0$). The examples of emerging markets are numerous: Video game industry in early 80's, PDA's in early 90's, currently smart phones.

Clearly, if $b^x = b^y = 0$ then in our model all compatibility regimes are equivalent in terms of monopolist's profit, equilibrium demands and prices. We can use this result to analyze the optimal compatibility choice in a situation where b^x and b^y are small, by investigating profits of the monopolist in the neighborhood of $(b^x, b^y) = (0,0)$. The comparison is summarized in the following proposition. Let us first define:

$$g_2(z) = \frac{2(1-B) - Bz(2s-1)}{zs(2s-B)}.$$

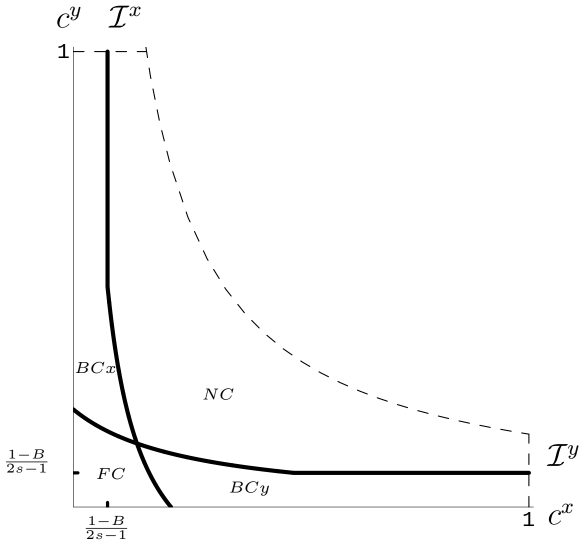
Proposition 5. *Assume that* b^x*,* b^y *are sufficiently small (close to zero). Then the following implications hold:*

$$\begin{aligned}
c^x < \frac{1-B}{2s-1} \quad &\text{or} \quad c^y < g_2(c^x) &&\Longrightarrow \quad NC \prec BCx \quad \text{and} \quad BCy \prec FC,\\
c^x > \frac{1-B}{2s-1} \quad &\text{and } c^y > g_2(c^x) &&\Longrightarrow \quad NC \succ BCx \quad \text{and} \quad BCy \succ FC,\\
c^y < \frac{1-B}{2s-1} \quad &\text{or} \quad c^x < g_2(c^y) &&\Longrightarrow \quad NC \prec BCy \quad \text{and} \quad BCx \prec FC,\\
c^y > \frac{1-B}{2s-1} \quad &\text{and } c^x > g_2(c^y) &&\Longrightarrow \quad NC \succ BCy \quad \text{and} \quad BCx \succ FC.
\end{aligned}$$

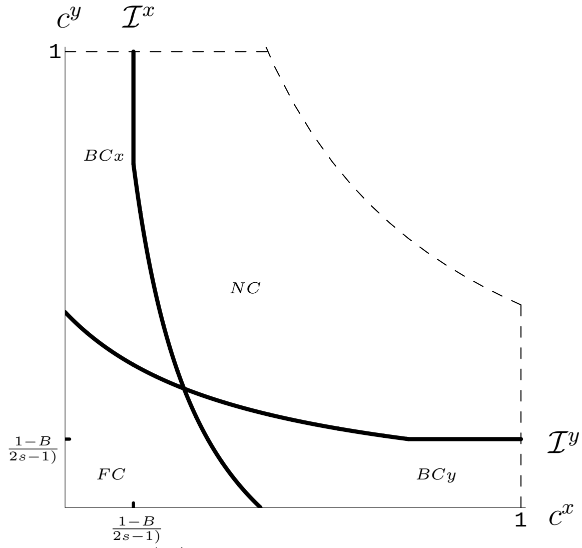
Similarly as before we can define indifference curves $\mathcal{I}^x$ and $\mathcal{I}^y$ which represent the indifference of the monopolist between making technologies compatible for agents of the respective type and making them incompatible.

$$\mathcal{I}^y = \begin{cases} \frac{1-B}{2s-1}, & \text{if } c^y \leq \frac{1-B}{2s-1}, \\ g_2(c^y), & \text{if } c^y > \frac{1-B}{2s-1}. \end{cases} \tag{11}$$

$\mathcal{I}^x$ is analogically defined. It is clear from the definition of the $\mathcal{I}^y$ and $g_2(z)$, that $\mathcal{I}^y$ is decreasing in B, s and is non-increasing in c^y (analogical result holds for $\mathcal{I}^x$). The indifference curves and optimal choice of compatibility regime are illustrated on the Figure 3 (the dashed line shows the area where Assumption 3 and Assumption 4 are satisfied).



(a) $s = 2.5$, $B = 0.7$



(b) $s = 1.5$, $B = 0.7$

Figure 3: Optimal choice of compatibility regimes at the emerging market

As is illustrated on the figure above, the backward compatibility for agents of type $i \in \{x, y\}$ is chosen if there are few new agents of this type. In particular, it is always optimal to make technologies compatible for i-agents if $c^i < \frac{1-B}{2s-1}$. If $c^i > \frac{1-B}{2s-1}$, then according to Proposition 5 there exist a cutoff value of c^i (which is the decreasing function of c^j), such that technologies will be incompatible for i-agents for all c^i above this value.

This result is due to the tradeoff between incentives of new i-agents and old j-agents. Consider for example BCy regime and assume that parameters other than c^y are fixed. If c^y is small then introduction of BCy regime has no significant effect on the incentives of old x-agents to buy the new technology (the direct negative effect is small). Indeed, it provides the agents who belong to the installed base with few additional connections. Their decision to purchase the new technology depends therefore on its characteristics and the demand of old y-agents, rather than on the number of new agents subscribed to it. As c^y increases, however, the access to the new agents starts to play more important role for the decision of the installed base to buy the new technology. In this case introduction of BCy leads to the significant reduction of the demand on behalf of old x-agents and the resulting feedback effect becomes also pronounced. In this case the monopolist is better off making technologies incompatible.

The threshold value of c^y is non-increasing in c^x. Indeed, if c^x is large the new y-agents would buy the technology even if it does not provide them an access to the installed base of x-agents. Hence introduction of compatibility will only moderately increase the demand on their behalf, while still discouraging the old agents from buying the new technology.

As follows from the Proposition 5, the decision of the principal to introduce backward compatibility for agents of type y does not depend on the compatibility of platforms for agents of type x. The intuition is similar to the case of mature markets. The monopolist ignores any implications which the change in demand of new x agents due to BCy has on the demand of old y agents. Indeed, this indirect effect is insignificant, because the installed base of y-agents is small.

Finally, we should observe that similarly to the case of mature market, the increase in the stand-alone benefits (B) or in the per-interaction benefits (s) shifts the indifference curve $\mathcal{I}^y$ downwards and $\mathcal{I}^y$ to the left. In other words the monopolist is less likely to make technologies compatible if the technological progress is revolutionary. This observation is formalized in the following corollary.

Corollary 2. *Consider some $c^x, c^y > 0$. Let $\bar{s} > 1$ and $\bar{B} \in (0, 1)$ satisfy Assumption 4 (where $b^x \to 0$ and $b^y \to 0$). If the monopolist makes technologies compatible for $s = \bar{s}$ and $B = \bar{B}$, then he will make technologies compatible for all $s \leq \bar{s}$ and $B \leq \bar{B}$.*

4.3 Asymmetric market

We define asymmetric market as a market where there is an installed base only on one side, that is, for example, a market where $b^x > 0$ and $b^y = 0$. Such asymmetry can exist because the market in question, although having characteristics of two-sided market, was treated by the monopolist as a "single-sided" business. One example is iPod/iTunes music platform. The (potential) two sides of the market in this case are users who download the music and publishers who provide it. However, as is documented in Evans et al. (2006, Ch. 6, pp. 213–244 and pp. 257), the two sides of the market in the case of iPod/iTunes platform do not interact with each other; in fact the publishers have no access to platform at all. Instead, Apple follows a vertically integrated strategy buying the music by paying publishers royalties and distributes this music to customers who want it. Also PDA's and smart phones evolved from a product which provided an integrated solution (hardware, operation system and applications) to the two-sided platforms, where applications and hardware are provided by the third-party developers (Evans et al. 2006, Ch. 6 and Ch. 9).

In this section we investigate which compatibility regime is going to be chosen at the market, where the monopolist who treated a platform as a single-sided business switches to the two-sided model. We assume therefore, that $b^x > 0$, $b^y = 0$, $c^x > 0$,

$c^y > 0$. Notice, $b^y = 0$ implies, that in terms of monopolist's profit, prices and demands, NC regime is equivalent to BCx regime and FC regime is equivalent to BCy regime. We will consider therefore only the choice between NC and BCy regimes. The logic, applied in previous cases holds also here. If we fix parameters s, B and b^x, then we can define an indifference curve (let us denote it $\mathcal{I}^y$), such that for the combination of (c^x, c^y) below this curve the monopolist would make technologies compatible and he would prefer them to be incompatible for the parameter range above this curve.

Proposition 6. *Let $b^y = 0$ and $b^x > 0$. Then for any c^x there exists a unique $\mathcal{I}^y = g_3(c^x)$ such that $NC \prec BCx$ if and only if $c^y < g_3(c^x)$.*

The indifference curve $\mathcal{I}^y$ and the optimal choice of compatibility regimes is illustrated on Figure 4.

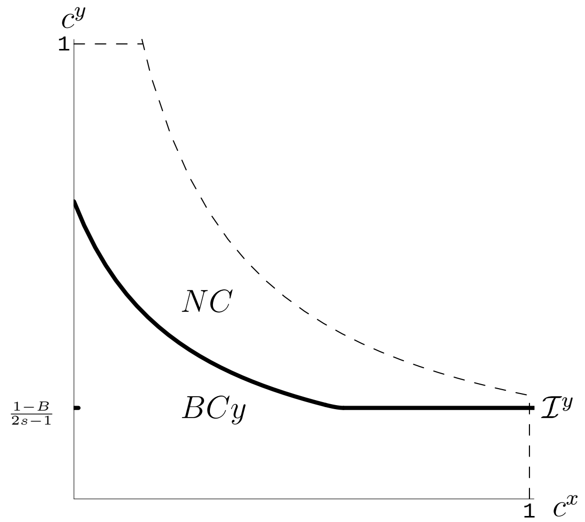


Figure 4: Optimal choice of compatibility regimes at the asymmetric market($s = 2$, $B = 0.4$, $b^x = 0.1$)

As the intuition discussed for the previous market structures suggests, the indifference curve $\mathcal{I}^y$ must be non-increasing in c^x. The tradeoff between NC and BCy regimes is driven by direct positive effect on the incentives of new agents of type y, price effect and the direct negative effect on the incentives of old agents of type x. If c^x is large, than the new agents of type y find the platform attractive even if it is not compatible with the old platform. In this case the positive direct effect of BCy is relatively unimportant, and hence NC will be chosen. If c^x is small, however, then introduction of BCy regime significantly improves the demand on behalf of new agents of type y — and this effect offsets the negative effect on demand of old agents of type x.

Improvement in technological characteristics (increase of s or B) of the new platform has similar effect on incentives of new y-agents as increase in c^x. On the other hand, any loss of demand on behalf of old agents of type x is more important for the monopolist's if s or B is large. Introduction of BCy regime not only discourages some agents from installed base to buy the new platform, but also the reduction in their demand triggers the reduction in the demand of new agents of type y. These two effects become more important as s or B increases. Hence, for larger s or B the monopolist is less willing to make technologies compatible.

Finally, if the size of installed base is large then the monopolist is less willing to introduce BCy regime. Indeed, if b^x is large compared to c^y, then the monopolist should be more concerned with the reduction in demand of the installed base, than with the increase in demand of new agents.

Although the intuition for comparative statics above does not depend on particular values of parameters, it is not possible to provide the analytical comparative statics for a general case. Therefore, let us assume, that the parameters of the model are such, that the interior solution is feasible.

Corollary 3. *Let $c^x, c^y > 0$, $\bar{b}^x > 0$, $\bar{s} > 1$, and $\bar{B} \in (0,1)$ be such that the interior solution is feasible. Then the indifference curve $\mathcal{I}^y = g_3(c^x)$ is downward sloping. Moreover, if the monopolist makes technologies compatible for some $s = \bar{s}$, $B = \bar{B}$, and $b^x = \bar{b}^x$, then (other things equal) he will make them compatible for all $s \leq \bar{s}$, $B \leq \bar{B}$, and $b^x \leq \bar{b}^x$.*

5 Discussion

We have identified three effects that backward compatibility for, say, agents of y type has on two-sided market (the argument is naturally the same for the backward compatibility for agents of x type). First there is a *direct effect* that is positive for the new y-agents (backward compatibility improves their incentives) and is negative for the old x-agents (backward compatibility discourages them from buying the new technology). Second, there is a *price effect* that is negative for the new y-agents. Finally, there are *negative feedback* effects. Namely, decrease in the demand of old agents of type x leads to the decrease in the demand of old agents of type y and to the decrease in the demand of new agents of type y. The negative feedback effects become more important if the technological progress is revolutionary (s and B are large), while the direct positive effect less so. Indeed, if the new platform is very advanced, then the new agents have large incentives to buy it even if it does not allow them to access the installed base of agents on the other side of the market. The compatibility therefore will bring only moderate improvement in their demand.

The tradeoff between direct effect, price effect and feedback effects determines the optimal compatibility choice. In particular, as follows from our analysis, the backward compatibility is more likely to be imposed on the market where the technological progress is moderate.[18] Further (other things equal), the compatibility for agents of type y is more likely to be imposed if their installed base is relatively small, the

[18]Note, that this argument does not rely on the costs of achieving compatibility, which are naturally higher if the platforms belong to the very distant generations of technology. Higher costs provide another reasons why the platforms should be incompatible if the technological progress is revolutionary.

growth rate of the installed base is moderate and the installed base of x-agents is small.

Our model provides several predictions about patterns of compatibility choice. On the emerging market, where technological progress is rapid and the entry of agents on both sides of the market is significant, we should often observe the subsequent generations of technologies being not compatible with each other. As, however, the pace of technological improvement slows down and the growth of installed base decelerates (the market becomes mature), we should expect some degree of compatibility between subsequent generations of technology. In particular, technologies are likely to be backward compatible for some side of the market if the installed base on this side of the market is relatively small. Technologies are likely to be fully compatible if the both sides of the market are symmetrically represented. Only if the technological progress is significant and the installed base on both sides of the market are very large should the technologies remain incompatible.

The predicted pattern of the compatibility regimes as a market develops from emerging to mature is nicely illustrated by the experience of video game console industry. The following discussion is adopted mainly from the Evans et al. (2006). The start of the video game console industry dates back to the earlier 70's. However, the industry was emerging at the slow pace. The leader of the industry, Atari, at the pick of it success sold only around 5 mln units of video game consoles. Moreover, the game industry crashed in 1983 due to the overproduction of poor quality games. The credit for the revival of the industry goes to Nintendo. Around 1983 Nintendo introduced its first console (Nintendo Entertainment System, NES) which has revolutionized the way how the video console business was done. Nintendo actively pursued a two-sided market strategy. It drafted licensing agreements with third party providers to ensure the quality of the games and the critical mass of the games for the new system. The sales of the NES and related games skyrocketed. It sold around 60 mln consoles world wide.

Ninetendo operated at a clearly emerging market, where the pace of technological growth was rapid and the installed base of users of consoles and game developers was relatively small. The future generations of Nintendo video game consoles were incompatible with the previous version.[19] Super NES (introduced in 1990) was incompatible with its predecessor NES; Nintendo 64 (introduced in 1996) was incompatible with Super NES and Game Cube (introduces in 2001) was incompatible with Nintendo 64.

Presently, in Japan, USA and Europe, the video game console market has reached its mature state. According to estimation of Nielsen, a market research company, 41.1% of US households own a game console and the rate of console penetration has slowed down[20]. In line with our predictions Nintendo made its new console, Wii (introduced in 2006) backward compatible with its predeccessor, Game Cube.

[19]Interesting enough, there are add-ons, unlicensed by Ninetendo, which allow to make subsequent generations of Nintendo consoles compatible. It indicates that Nintendo decided to make platforms incompatible not because it was technically impossible, but because it was more profitable strategy. The information about backward compatibility is taken from `en.wikipedia.org/wiki/Backward_compatibility.`

[20]See report of The Nielsen Company (2006)

Our analysis also indicates how the compatibility of platforms should evolve on the asymmetric market where a monopolist, who previously treated his market as a one-sided business decides to disintegrate and to embrace a two-sided model. Following Section 4.3 let us assume that there is an installed base only on the x side of the market and the growth rate of the market are c^x and c^y. Then we would expect subsequent generations of platforms be compatible for agents of type y, if c^y is small and/or if the pace of technological progress is moderate.

To illustrate this prediction, consider the case of Palm company.[21] Palm started as a software company but soon integrated in a hardware. Although it did not produced the hardware itself, it controlled all stages of the process and treated the involved firms as subcontractors. PalmPilot, produced in 1996, was a hardware with integrated operation system. However, in late 1997 Palm switched to a two-sided model. It has concentrated on the development of Palm OS operation system, which it was licensing to the hardware makers, such as Sony, Kyocera, Nokia, Handspring etc. (y agents, in the terminology of our model). On the other hand, to ensure the popularity of Palm OS, Palm has intensively courted the developers of applications (the x side of the market) from the time of introduction of Palm Pilot. It already had significant installed base of third party developers when it decided to switch to the two-sided model. The efficient courting strategy ensured that this base was growing fast. However, due to some management failures and the market trends, Palm had less success in ensuring the cooperation of third party providers of hardware. Sony, for example, has stopped selling PDA's which run Palm OS. Handspring was purchased by PalmOne (a hardware company, independent from PalmSource, provider of Palm OS). In line with our prediction, Palm, eager to improve attractiveness of its operation system for the hardware developers, made it backward compatible. Any version of Palm OS, installed on hardware device, is not only able to run the applications, written for this version but also applications written for the older versions of the operation system.

6 Conclusion

In this paper we developed a theory of compatibility choice at two-sided market. This theory is an important contribution to the literature on two-sided markets which up to now did not devoted much attention to the issues of compatibility.

Our first important result is that the monopolist will never choose partial compatibility. He will either make technologies incompatible or will make them compatible to the extent that agents, who interact using different platforms, can enjoy maximal possible network benefits. This result allows us to concentrate our analysis on four extreme compatibility regimes: full compatibility, incompatibility and backward compatibility for each side of the market.

We showed that the tradeoff which is in the heart of monopolist's decision to make technologies compatible, is the tradeoff between demand of new agents on one

[21] The example is taken from Evans et al. (2006, Ch. 6 and Ch. 9). The data about compatibility of Palm OS is available at www.access-company.com/developers/documents/docs/palmos.

side of the market and demand of the old agents on the other side of the market. In particular, if the monopolist introduces backward compatibility for, say, users, he encourages new users to buy the new platform but discourages the old sellers to do so (*direct effect*). The decrease in the demand of old sellers triggers the decrease in demand of old users and of the new users (*feedback effect*). Finally, compatibility leads to higher prices for new users, which has a negative effect on their demand (*price effect*). The tradeoff between these effects determines which compatibility regime will be chosen in equilibrium.

Investigating different market structures (mature market, emerging market and asymmetric market) we characterized the choice of compatibility in terms of primitives of the model. In particular, we showed, that the compatibility for users will be imposed if the proportion of new users is relatively small, installed base of sellers and users is relatively small and the technological progress is moderate. We illustrate our predictions about the pattern of compatibility choice with two examples.

Our model can be modified in several ways. First, we assume that both sides of the market are symmetric in terms of per-interaction benefits. This is not necessarily the case on two-sided markets. We could modify the model by introducing some asymmetry between agents. This modification, however, would not change the underlying intuition and therefore the basic results.

Another assumption which we impose is that the quality of interaction between users and sellers is fixed and depends on the lowest technology which enables this interaction. For some markets other technological assumptions can be more realistic; for example, the quality of interaction may be determined by the best of two technologies. It would be useful to see how the choice of compatibility regime depends on technological assumptions.

A Appendix: Proofs

Proof of Lemma 1. Taking the second derivatives, we obtain the *Hessian matrix*:

$$H = \begin{bmatrix} -b^x H''(m^x) & (s-1)b^x b^y & 0 & (s-\gamma^y)b^x c^y \\ (s-1)b^x b^y & -b^y H''(m^y) & (s-\gamma^x)b^y c^x & 0 \\ 0 & (s-\gamma^x)b^y c^x & -c^x H''(n^x) & sc^x c^y \\ (s-\gamma^y)b^x c^y & 0 & sc^x c^y & -c^y H''(n^y) \end{bmatrix} \tag{12}$$

In order to obtain concavity, we need to show that $(-1)^j D_j > 0$, where D_j is the leading principal minor of order $j = 1, 2, 3, 4$.

Let us now fix some $\gamma^x, \gamma^y \in [0, 1]$. We will show that the following three conditions are sufficient for concavity:

(i') $\Delta > 0$;

(ii') $\Delta^2 > (s-1)^2 b^x b^y + (s-\gamma^x)^2 b^y c^x$; and

(iii') $\Delta^4 + \Gamma^2 b^x b^y c^x c^y > [(s-1)^2 b^x b^y + (s-\gamma^x)^2 b^y c^x + (s-\gamma^y)^2 b^x c^y + s^2 c^x c^y]\Delta^2$, where $\Gamma = s(s-1) - (s-\gamma^x)(s-\gamma^y)$. Note that these conditions reduce to (i)–(iii) when $\gamma^x = \gamma^y = 0$.

Because $\Delta > 0$, then $H''(z) > 0$ for all $z \in [0,1]$. Consequently, $D_1 < 0$. Condition (ii') implies that $\Delta^2 > (s-1)^2 b^x b^y$ and, thus, $D_2 > 0$. Moreover, it follows from condition (ii') that $\Delta^2 > (s-\gamma^x)^2 b^y c^x$ and

$$\begin{aligned}
&\Delta^2[\Delta^2 - (s-1)^2 b^x b^y - (s-\gamma^x)^2 b^y c^x] = \\
&= [\Delta^2 - (s-1)^2 b^x b^y]\,[\Delta^2 - (s-\gamma^x)^2 b^y c^x] - (s-1)^2 b^x b^y (s-\gamma^x)^2 b^y c^x \leq \\
&\leq [\Delta H''(m^x) - (s-1)^2 b^x b^y]\,[\Delta H''(n^x) - (s-\gamma^x)^2 b^y c^x] - (s-1)^2 b^x b^y (s-\gamma^x)^2 b^y c^x = \\
&= \Delta[\Delta H''(m^x) H''(n^x) - (s-1)^2 b^x b^y H''(n^x) - (s-\gamma^x)^2 b^y c^x H''(m^x)]
\end{aligned}$$

Now,

$$-\frac{D_3}{b^x b^y c^x} = H''(m^x) H''(m^y) H''(n^x) - (s-1)^2 b^x b^y H''(n^x) - (s-\gamma^x)^2 b^y c^x H''(m^x)$$

and the inequality $D_3 < 0$ follows from the fact that $H''(m^y) \geq \Delta$.

In order to prove that $D_4 > 0$, consider first the matrix H when $H''(m^x) = H''(m^y) = H''(n^x) = H''(n^y) = \Delta$. It follows from conditions (i')–(iii') that this matrix is negative definite. Thus, we obtain similar conditions as (ii') also for other principal minors of order 3:

$$\Delta^2 \geq (s-1)^2 b^x b^y + (s-\gamma^y)^2 b^x c^y, \qquad \Delta^2 \geq (s-\gamma^x)^2 b^y c^x + s^2 c^x c^y,$$
$$\Delta^2 \geq (s-\gamma^y)^2 b^y c^x + s^2 c^x c^y.$$

By the same procedure as above, we may show that all principal minors of order 3 of matrix H are non-negative. Direct computation reveals that

$$\begin{aligned}
\frac{D_4}{b^x b^y c^x c^y} &= H''(m^x) H''(m^y) H''(n^x) H''(n^y) + \Gamma^2 b^x b^y c^x c^y - \\
&- [(s-1)^2 b^x b^y H''(n^x) H''(n^y) + (s-\gamma^x)^2 b^y c^x H''(m^x) H''(n^y) + \\
&+ (s-\gamma^y)^2 b^x c^y H''(m^y) H''(n^x) + s^2 c^x c^y H''(m^x) H''(m^y)].
\end{aligned}$$

Now, it can be easily shown that the value of D_4 does not increase when subsequently substitute $H''(n^y) = \Delta$, $H''(n^x) = \Delta$, $H''(m^y) = \Delta$, and $H''(m^x) = \Delta$. At the end we obtain a positive expression due to (iii').

Now it remains to show that conditions (i)–(iii) imply conditions (i')–(iii'). Condition (i') is identical to (i) and clearly, the condition (ii') follows from (ii), since its right-hand side is decreasing in γ^x. Now, let is rewrite the condition (iii') as $\Delta^4 + \Gamma^2 b^x b^y c^x c^y - [(s-1)^2 b^x b^y + (s-\gamma^x)^2 b^y c^x + (s-\gamma^y)^2 b^x c^y + s^2 c^x c^y]\Delta^2 > 0$. It's derivative with respect to γ^x is $2b^y c^x \left[b^x c^y s(s-1)(s-\gamma^y) + (s-\gamma^x)\big(\Delta^2 - (s-\gamma^y)^2 b^x c^y\big)\right]$, which is non-negative. Similarly, we may show that the derivative with respect to γ^y is non-negative. □

Proof of Lemma 2. The uniqueness of the maximizer follows from concavity. Taking the first derivative of the profit with respect to m^x we obtain

$$\frac{\partial \Pi}{\partial m^x} = b^x[-H'(m^x) + (s-1) b^y m^y + (s-\gamma^y) c^y n^y].$$

If follows from the definition of H that $H'(z) = \frac{1}{2}G(z) + \frac{1}{2}zG'(z)$. By Assumption 1 we have $\lim_{z\to 1^-} G'(z) = +\infty$. Thus, $\lim_{z\to 1^-} H'(z) = +\infty$ and $\partial\Pi/\partial m^x|_{m^x=1} < 0$. Moreover, when $G(z) = \theta$, then $2H'(z) = \theta + F(\theta)/F'(\theta)$ and $2H'(0) = \underline{\theta} + \lim_{\theta\to\underline{\theta}^+} F(\theta)/F'(\theta)$, which is negative by Assumption 1. Therefore, $\partial\Pi/\partial m^x|_{m^x=0} > 0$. This implies, that $0 < m^x < 1$ in maximum. Such m^x then satisfies the first-order condition $\partial\Pi/\partial m^x = 0$. The proofs for m^y, n^x, and n^y are analogous. □

Proof of Proposition 1. The proposition follows from part (i) of Lemma 4. □

Proof of Proposition 2. The proposition follows immediately from Lemma 4, parts (ii) and (iii). □

Lemma 4. *For any $\gamma^y \in [0,1]$ the following statements hold:*

(i) *If the monopolist is free to choose any $\gamma^x \in [0,1]$, he would choose either $\gamma^x = 0$ or $\gamma^x = 1$.*

(ii) *If $m^y \leq \frac{1}{2}$ in optimum for $\gamma^x = 0$, then the monopolist would choose $\gamma^x = 1$.*

(iii) *If $m^y \geq \frac{1}{2}$ in optimum for $\gamma^x = 1$, then the monopolist would choose $\gamma^x = 0$.*

Proof of Lemma 4. Taking the partial derivative of monopolist's profit (9), we obtain

$$\frac{\partial\Pi}{\partial\gamma^x} = b^y c^x n^x (1 - 2m^y). \tag{13}$$

For $\gamma^x \in [0,1]$, let $\tilde{m}^x(\gamma^x)$, $\tilde{m}^y(\gamma^x)$, $\tilde{n}^x(\gamma^x)$, and $\tilde{n}^y(\gamma^x)$ be the solution of the first-order conditions from Lemma 2 and let $\tilde{\Pi}(\gamma^x) = \Pi\big(\tilde{m}^x(\gamma^x), \tilde{m}^y(\gamma^x), \tilde{n}^x(\gamma^x), \tilde{n}^y(\gamma^x)\big)$. Using the *Envelope Theorem* we obtain that $\mathrm{d}\tilde{\Pi}(\gamma^x)/\mathrm{d}\gamma^x = b^y c^x \tilde{n}^x(\gamma^x)\big(1 - 2\tilde{m}^y(\gamma^x)\big)$.

Now we will show that if $\tilde{m}^y(\bar{\gamma}^x) \geq \frac{1}{2}$ for some $\bar{\gamma}^x > 0$, then $\tilde{m}^y(\gamma^x) > \frac{1}{2}$ for all $\gamma^x \in [0, \bar{\gamma}^x)$. Using the *Implicit Function Theorem* for the first-order conditions, we obtain that $\tilde{m}^y(\gamma^x)$ is continuous and differentiable with derivative

$$\begin{aligned}\frac{\mathrm{d}\tilde{m}^y}{\mathrm{d}\gamma^x}(\gamma^x) = \frac{b^x b^y c^x c^y}{2D_4} c^x \big[& b^y(1-2m^y)\big((s-\gamma^x)H''(m^x)H''(n^y) + (s-\gamma^y)\Gamma b^x c^y\big) + \\ & +2n^x\big(-H''(m^x)H''(n^x)H''(n^y) + s^2 c^x c^y H''(m^x) + (s-\gamma^y)^2 b^x c^y H''(n^y)\big)\big],\end{aligned} \tag{14}$$

where D_4 is the determinant of the *Hessian matrix* H defined in (12) and $\Gamma = s(s-1) - (s-\gamma^x)(s-\gamma^y)$. Observe that the coefficient at n^x is actually a third minor of matrix H multiplied by a positive factor and is, thus, negative. In addition, as $(s-\gamma^x)(s-\gamma^y)^2 \geq -(s-\gamma^y)\Gamma$ and $H''(m^x)H''(n^y) > (s-\gamma^y)^2 b^x c^y$, then the coefficient at $(1-2m^y)$ is positive. Therefore, if $\tilde{m}^y(\gamma^x) \geq \frac{1}{2}$, then $\mathrm{d}\tilde{m}^y(\gamma^x)/\mathrm{d}\gamma^x < 0$, which yields the desired statement.

Consequently, if $\tilde{m}^y(1) \geq \frac{1}{2}$, then $\tilde{m}^y(\gamma^x) > \frac{1}{2}$ for all $\gamma^x \in [0,1)$. Thus, $\tilde{\Pi}(\gamma^x)$ is decreasing on $[0,1]$ and the monopolist would choose $\gamma^x = 0$. This proves (iii). On the other hand, if $\tilde{m}^y(0) \leq \frac{1}{2}$, then it is not possible that $\tilde{m}^y(\gamma^x) \geq \frac{1}{2}$ for some $\gamma^x \in (0,1]$. Thus, $\tilde{m}^y(\gamma^x) < \frac{1}{2}$ for all $\gamma^x \in (0,1]$ and $\tilde{\Pi}(\gamma^x)$ is decreasing on $[0,1]$. Therefore, the monopolist would choose $\gamma^x = 1$, which proves (ii). Finally, if $\tilde{m}^y(0) > \frac{1}{2} > \tilde{m}^y(1)$, then there exists unique $\tilde{\gamma}^x \in (0,1)$ such that $\tilde{m}^y(\tilde{\gamma}^x) = \frac{1}{2}$. Then also $\tilde{m}^y(\gamma^x) > \frac{1}{2}$ when $\gamma^x \in [0, \tilde{\gamma}^x)$ and $\tilde{m}^y(\gamma^x) < \frac{1}{2}$ when $\gamma^x \in (\tilde{\gamma}^x, 1]$. Thus, $\tilde{\Pi}(\gamma^x)$ is decreasing on $[0, \tilde{\gamma}^x)$ and increasing on $(\tilde{\gamma}^x, 1]$. This means, that the monopolist's profit can achieve its maximum only when $\gamma^x = 0$ or $\gamma^x = 1$, which together with the previous statements proves (i). □

Proof of Proposition 3. The derivatives can be obtained from the first-order conditions in Lemma 2 using the *Implicit Function Theorem.* The derivative $\mathrm{d}m^y/\mathrm{d}\gamma^x$ is computed in the proof of Lemma 4 and is given by (14). The coefficients β_1 and β_2 can be written as $\beta_1 = \tilde{\beta}_1 b^x b^y c^x c^y/(2D_4)$, and $\beta_2 = \tilde{\beta}_2 b^x b^y c^x c^y/(2D_4)$, where the expressions for $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are summarized in the Table 4 in Appendix B. Using analogous arguments as in the proof of Lemma 4, we can verify that $\beta_1 > 0$ and $\beta_2 < 0$. The effect on profit also follows from that proof. □

Proof of Lemma 3. To prove the first statement, consider a case where monopolist sets prices so that the demands are given by $m^x = 0, m^y \in [0,1], n^y \in [0,1]$ and $n^x \in [0,1]$. This allocation is never an equilibrium. Indeed, the monopolist can marginally decrease price A_0^x to gain a positive demand on behalf of old x-agents. Since the prices for other groups of agents remain unchanged, the profit of the monopolist will be strictly larger.

The proof of the second and the third statement results from the fact that, given fixed m^y, n^x and n^y, the profit of the monopolist is quadratic, concave function of A_0^x. Hence, if the solution of the first-order condition with respect to A_0^x leads to $m^{x*} \in [0,1]$, then $m^x = m^{x*}$ maximizes monopolist's profit. If $m^{x*} > 1$, then the profit of the monopolist increases in $m^x = 1$ and therefore on the interval $m^x \in [0,1]$ the profit is maximized if $m^x = 1$. □

Corollary 4. *Assume that $i_k, j_k \in \{0,1\}$ are such that $i_k \leq j_k$ for $k = 1,2,3,4$ and at least one of these inequalities is strict. If both equilibria $E_{i_1 i_2 i_3 i_4}$ and $E_{j_1 j_2 j_3 j_4}$ feasible, then monopolist's profit in equilibrium $E_{i_1 i_2 i_3 i_4}$ is higher.*

Proof of Corollary 4. The proof follows directly from Lemma 3. □

Proof of Proposition 4. We provide comparison between NC and BCx regimes. The comparison of other regimes is done by analogously.

Before proceeding with comparison of NC and BCx regimes we need to describe equilibrium allocations for each regime. Here we will derive the equilibrium allocations for NC regime. The equilibrium allocations for BCx regimes are derived by analogy. Let us denote

$$h_1 = \frac{2-B}{Bs + b^x(s-1)(2s-2+B)}, \qquad h_2 = \frac{2-B-Bb^x s}{b^x(s-1)(2s-2+B)},$$
$$h_3 = \frac{2-B}{[B+2b^x(s-1)](s-1)}, \qquad h_4 = \frac{2-B(1-b^x+b^x s)}{2b^x(s-1)^2}.$$

Further, let $b^y = h_5(b^x)$ be implicitly given by the equation $b^x = \frac{2-b^y-B[1+b^y(s-1)]}{(2-b^y)b^y(s-1)^2}$. Obviously, functions h_i, $i = 1, \ldots, 5$ are decreasing in b^x.

The feasibility conditions for each equilibrium candidates are summarized in Table 6 in Appendix B in the first and third column for NC and BCx regimes respectively. Notice, that each allocation $E_{i_1 i_2 i_3 i_4}$ is feasible only for some range of parameters. The relevant feasibility conditions result from the requirement that in equilibrium demands must be such, that $m_i \in [0,1]$ and $n_i \in [0,1]$ for any $i \in \{x, y\}$. With some abuse of notation and terminology, we will say that allocation $E_{i_1 i_2 i_3 i_4}$ *dominates* allocation $E_{j_1 j_2 j_3 j_4}$ ($E_{j_1 j_2 j_3 j_4} \prec E_{i_1 i_2 i_3 i_4}$) if the profit of the monopolist is larger in the former case, than in the later.

We call conditions which ensure that a particular allocation is an equilibrium *optimality conditions*. The optimality conditions for NC and BCx regimes are summarized in the second and the fourth column of Table 6 respectively.

Consider NC regime, allocation E_{1000}:

$$m^x = 1, \qquad n^x = \frac{-2b^x b^y (s-1)s - 2b^x c^y s^2 - B(1 + b^y s + c^y s)}{2(-1 + b^y c^x s^2 + c^x c^y s^2)},$$
$$m^y = \frac{-B(1 + c^x s) + b^x(2 - 2s - 2c^x c^y s^2)}{2(-1 + b^y c^x s^2 + c^x c^y s^2)}, \; n^y = \frac{B(-1 - c^x s) - 2b^x s(1 - b^y c^x s)}{2(-1 + b^y c^x s^2 + c^x c^y s^2)}.$$

It is easy to verify that given $c^x \to 0$, $c^y \to 0$ this allocation is feasible if

$$b^x < \frac{2 - B}{2s} \quad \text{and} \quad b^y < \frac{2 - B}{s[B + 2b^x(s-1)]}.$$

From Table 6, allocation E_{0000} is feasible if $b^y < \min[h_1, h_2]$. It is clear, that $h_1 > \frac{2-B}{s(B-2b^x+2b^x s)}$. Further, h_1 and h_2 are decreasing in b^x and $h_1 = h_2$ if $b^x = \frac{2-B}{2s}$. Hence, whenever E_{1000} is feasible, E_{0000} is also feasible. By Corollary 4, $E_{1000} \prec E_{0000}$. Similar argument allows to establish, that allocations E_{1001}, E_{0100}, E_{0110}, E_{1100}, E_{1110}, and E_{1101} never occur in equilibrium.

Therefore, the set of equilibrium candidates is limited to nine allocations E_{0000}, E_{0010}, E_{0011}, E_{0001}, E_{1010}, E_{1011}, E_{0111}, E_{0101}, and E_{1111}. Using Lemma 3 we can immediately see, that whenever E_{0000} is feasible, it is optimal. By Corollary 4, $E_{0011} \prec E_{0010}$ and $E_{0011} \prec E_{0001}$ whenever the allocations are simultaneously feasible. Further, $E_{1010} \prec E_{0010}$, $E_{0101} \prec E_{0001}$, $E_{1011} \prec E_{1010}$, $E_{0111} \prec E_{0101}$, $E_{0111} \prec E_{0011}$, and $E_{1011} \prec E_{0011}$, whenever allocations are simultaneously feasible. Combining these results, we establish that the feasibility conditions divide the whole range of parameters into nine domains as characterized in the Table 6.

Having the characterization of equilibrium allocation, we now proceed with comparison of profits in BCx and NC regimes. As follows from Table 6, if we fix B and s then the whole range of parameters $b^x \in [0,1]$, $b^y \in [0,1]$ can be divided into several domains where for each compatibility regime a particular allocation is optimal. These domains are illustrated on Figure 5 in Appendix B. Note, that the dashed line shows the range of parameters where Assumptions 3 and 4 are satisfied. In the proof of the proposition we will assume that these conditions are satisfied for all domains. This assumption is without loss of generality, because if the conditions are not satisfied for some domain, then this domain is not feasible.

To prove the proposition we need to compare profits of the principal in NC and BCx regimes in each of those domains. The domains and corresponding equilibrium allocations are summarized in Table 1. We analyze each domain separately.

domain	feasibility condition	type of allocation NC	BCx
a	$b^y < \min[h_1, h_2, h_5]$	E_{0000}	E_{0000}
b	$h_5 < b^y < h_1$	E_{0000}	E_{0010}
c	$h_1 < b^y < h_5$	E_{0010}	E_{0000}
d	$\max[h_1, h_5] < b^y < \min[h_2, h_3]$	E_{0010}	E_{0010}
e	$b^y > h_3, b^x < \frac{2-B}{2s}$	E_{1010}	E_{1010}
f	$b^y > h_3, \frac{2-B}{2s} < b^x < \frac{2-B}{2(s-1)}$	E_{1011}	E_{1011}
g	$\max[h_2, h_5] < b^y < \min[h_3, h_4]$	E_{0011}	E_{0011}
h	$\max[h_1, h_2] < b^y < \min[h_4, h_5]$	E_{0011}	E_{0001}
i	$h_2 < b^y < \min[h_1, h_4]$	E_{0001}	E_{0001}
j	$h_4 < b^y < \frac{2-B}{2s}$	E_{0101}	E_{0101}
k	$\max[\frac{2-B}{2s}, h_4] < b^y < \frac{2-B}{2s-1}$	E_{0111}	E_{0101}
l	$\max[\frac{2-B}{2s-1}, h_4] < b^y < \frac{2-B}{2(s-1)}$	E_{0111}	E_{0111}
m	$\min[b^y, b^x] > \frac{2-B}{2(s-1)}$	E_{1111}	E_{1111}

Table 1: Equilibrium allocations on the mature market

$\triangleright$ *Domains* **a**, **d**, **g**, **i**

Consider first domain **a**. In this domain the interior solution (allocation of the type E_{0000}) is an equilibrium both in NC and BCx regimes. Given $c^x \to 0$, $c^y \to 0$, the equilibrium prices and allocations in NC regime are as follows:

$$m^x = \frac{B}{2} \cdot \frac{1 + b^y(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^x = \frac{B}{2} \cdot \frac{1 + b^x b^y (s-1) + b^y s}{1 - b^x b^y (s-1)^2},$$

$$m^y = \frac{B}{2} \cdot \frac{1 + b^x(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^y = \frac{B}{2} \cdot \frac{1 + b^x b^y (s-1) + b^x s}{1 - b^x b^y (s-1)^2},$$

$$A_0^x = A_0^y = A_1^x = A_1^y = \frac{B}{2}.$$

In BCx regime the equilibrium allocation and prices are

$$m^x = \frac{B}{2} \cdot \frac{1 + b^y(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^x = \frac{B}{2} \cdot \frac{1 + b^y(s-1)}{1 - b^x b^y (s-1)^2} + \frac{b^y}{2},$$

$$m^y = \frac{B}{2} \cdot \frac{1 + b^x(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^y = \frac{B}{2} \cdot \frac{1 + b^x b^y (s-1) + b^x s}{1 - b^x b^y (s-1)^2},$$

$$A_0^x = A_0^y = A_1^y = \frac{B}{2}, \ A_1^x = \frac{B + b^y}{2}.$$

Calculating the respective profits of the monopolist Π^{NC} and Π^{BCx}, it is straightforward to establish, that $\Pi^{NC} < \Pi^{BCx}$ if and only if $B < \frac{1-b^x b^y (s-1)^2}{1+b^x(s-1)}$ or equivalently $b^y < g_1(b^x)$. The same result holds for domains **d**, **g** and **i**.

▷ *Domains* **e** *and* **f**

Consider now domain **e**. Here the equilibrium allocation is of the type E_{1011} in NC and BCx regimes. Comparing the respective profits, we establish, that

$$\Pi^{NC} < \Pi^{BCx} \Longleftrightarrow -1 + B - c^x + 2b^x(s-1) + 2c^x s < 0.$$

Given c^x and c^y close to zero the above inequality implies that $NC \prec BCx$ if and only if $b^x < \frac{1-B}{2(s-1)}$. The same result holds in domain **f**.

▷ *Domains* **j**, **l** *and* **m**

In domains **j**, **l** and **m**, NC regime is clearly preferred to BCx regime. Here in both compatibility regimes $m^y = 1$ in equilibrium. Therefore, whether technologies compatible or not, new x-agents can interact with old y-agents since all of them are subscribed to the new technology. BCx regime therefore has no positive effect on the incentives of new x-agents.

▷ *Domain* **k**

In this domain the equilibrium allocation in BCx and NC regimes are of the type E_{0101} and E_{0111} respectively. Comparing the profits of the monopolist we receive the following equivalence:

$$\Pi^{NC} > \Pi^{BCx} \Longleftrightarrow 4 + B^2 + (b^y)^2(2s-1)^2 - 8b^y s + B[-4 + b^y(4s-2)] < 0.$$

The inequality above is satisfied if $b_1 < b^y < b_2$, where

$$b_{1,2} = \frac{B + 4s - 2Bs \mp 2\sqrt{4s - 1 + B - 2Bs}}{1 - 4s + 4s^2}.$$

Notice, that $4s - 1 + B - 2Bs > 0$ for any $s > 1$, $B \in (0,1)$. In addition, domain **k** is feasible if $\frac{2-B}{2s} < b^y < \frac{2-B}{2s-1}$. It is easy to show that $b_1 < \frac{2-B}{2s}$ and $b_2 > \frac{2-B}{2(s-1)}$. Hence, for any $\frac{2-B}{2s} < b^y < \frac{2-B}{2s-1}$, NC regime is preferred to the BCx regime.

▷ *Domains* **c**, **b** *and* **h**

Consider now domain **c**. Here the optimal allocation in NC regime is of the type E_{0010} and in BCx regime is of the type E_{0000}. Comparing the respective profits, we receive, that $\Pi^{BCx} \prec \Pi^{NC}$ if $\varphi(b^x)/\psi(b^x) > 0$, where

$$\psi(b^x) = -1 + b^x b^y (s-1)^2 + b^y c^x (s-1)^2 + c^x c^y s^2 + b^x c^y s^2.$$

Due to Assumption 4, $\psi(b^x) > 0$. We do not list the expression for $\varphi(b^x)$ here because this is a rather complicated. Function $\varphi(b^x)$ is quadratic in b^x and the coefficient at $(b^x)^2$ is $-4B(b^y)^2(s-1)^3 - (b^y)^2[4 + (b^y)^2](s-1)^4 < 0$. Hence, $\varphi(b^x) > 0$ if and only if $b_1 < b^x < b_2$, where $b_{1,2}$ are roots of the quadratic equation $\varphi(b^x) = 0$.

$$b_{1,2} = \frac{[4 + (b^y)^2](s-1) + B[(b^y)^2(s-1)^2 + 4 - 2s - b^y(s-1)(2s-1) \mp 2k(1 + b^y(s-1))]}{b^y[4B + (4 + (b^y)^2)(s-1)](s-1)^2}$$

where $k = \sqrt{-1 + B - Bs + (2 + b^y)s - b^y s^2}$. In addition, domain **c** is feasible if $h_1 < b^y < h_5$, which is equivalent to

$$\frac{-2 + b^y + B[1 + b^y(s-1)]}{(-2 + b^y)b^y(s-1)^2} < b^x < \frac{2 - B - Bb^y s}{b^y(s-1)(2s - 2 + B)}. \tag{15}$$

Given $b^x \geq 0$, the inequalities above are satisfied if and only if $b^y < \frac{2-B}{s}$. It is easy to show, that $b_1 < \frac{-2+b^y+B[1+b^y(s-1)]}{(-2+b^y)b^y(s-1)^2}$ and $b_2 > \frac{2-B-Bb^y s}{b^y(s-1)(2s-2+B)}$ for any $b^y < \frac{2-B}{s}$. Hence, $\varphi(b^x) > 0$ for any $h_1 < b^y < h_5$, which implies that $BCx \prec NC$.

By analogy we can show, that $BCx \prec NC$ in domain **h** and $NC \prec BCx$ in domain **b**. Summarizing the results, we receive that $BCx \prec NC$ if $b^x < \frac{1-B}{2(s-1)}$ or $b^y < g_1(b^x)$. □

Proof of Proposition 5. Similarly as in the proof of Proposition 4, we provide comparison between NC and BCx regimes. The comparison of other regimes is done by analogy.

Before comparing NC and BCx regimes we need to describe equilibrium allocations for each regime. Here we will derive the equilibrium allocations for NC regime. The equilibrium allocations for BCx regimes are derived by analogy. For that, let us denote

$$h_1 = \frac{2-B}{Bs + 2c^x s^2}, \qquad h_2 = \frac{2 - B - Bc^x s}{2c^x s^2}, \qquad h_3 = \frac{2 - B - Bc^x(s-1)}{2c^x s(s-1)}.$$

The feasibility conditions for each equilibrium candidates are summarized in Table 7 (in Appendix B) in the first and third column for NC and BCx regimes respectively. The optimality conditions for NC and BCx regimes are summarized in the second and the fourth column of Table 7 respectively.

We can directly eliminate the following allocations from the set of equilibrium candidates: E_{1000},E_{0100}, E_{1100}, E_{0010} and E_{0001}. Indeed, as demonstrated in Table 7, all those allocations are feasible if and only if allocation E_{0000} is feasible, which (according to Lemma 3) always generates larger profit. The same argument readily allows to establish that $E_{1011} \prec E_{1010}$, $E_{0111} \prec E_{0101}$, $E_{1110} \prec E_{1010}$, and $E_{1101} \prec E_{0101}$. Further notice, that $h_1 = h_2 = \frac{2-B}{2s}$ if $c^x = \frac{2-B}{2s}$. Hence, allocation E_{0000} is feasible whenever $\frac{2-B}{2s} > \max[c^x, c^y]$. Since feasibility conditions for E_{0011} require $\frac{2-B}{2s} > \max[c^x, c^y]$, we conclude that $E_{0011} \prec E_{0000}$. Finally, allocation E_{1001} is feasible if $c^y < \min[h_1, \frac{2-B}{2s}]$. Since $\min[h_1, \frac{2-B}{2s}] < \min[h_1, h_2]$, allocation E_{0000} is feasible whenever E_{1001} is feasible and $E_{1001} \prec E_{0000}$ by Lemma 3.

This leaves us with four candidates for equilibrium: E_{0000}, E_{1010}, E_{0101} and E_{1111}. Whenever allocation E_{0000} is feasible, it is optimal. Moreover, by Lemma 3, whenever E_{1010} and E_{1111} are simultaneously feasible, $E_{1111} \prec E_{1010}$ (the symmetric argument holds for allocation E_{0101}). Therefore, the whole range of feasible parameters c^x and c^y is divided into four domains, as summarized in Table 7.

Having the characterization of equilibrium allocation, we now proceed with comparison of profits in BCx and NC regimes. As follows from Table 7, if we fix B and s then the whole range of parameters $b^x \in [0,1]$, $b^y \in [0,1]$ can be divided into several domains where for each compatibility regime a particular allocation is optimal. These domains are illustrated on Figure 6 in Appendix B. Note, that the dashed line shows the range of parameters where Assumptions 3 and 4 are satisfied. In the proof of the proposition we will assume that these conditions are satisfied for all domains. This assumption is without loss of generality, because if the conditions are not satisfied for some domain, then this domain is not feasible.

To prove the proposition we need to compare profits of the principal in NC and BCx regime in each of the domains. The domains and corresponding equilibrium allocations for NC and BCx regimes are summarized in Table 2. We analyze each domain separately.

domain	feasibility condition	type of allocation NC	BCx
a	$c^y < \min[h_1, h_2]$	E_{0000}	E_{0000}
b	$h_1 < c^y, c^x < \frac{2-B}{2s}$	E_{1010}	E_{1010}
c	$c^y > \frac{2-B}{2s}, \frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$	E_{1111}	E_{1011}
d	$c^y > \frac{2-B}{2s}, c^x > \frac{2-B}{2(s-1)}$	E_{1111}	E_{1111}
e	$h_3 < c^y < \frac{2-B}{2s}$	E_{0101}	E_{0101}
f	$h_2 < c^y < \min[h_3, \frac{2-B}{2s}]$	E_{0101}	E_{0001}

Table 2: Equilibrium allocations on the emerging market

▷ *Domains* **d** *and* **e**

We can immediately establish, that $BCx \prec NC$ in domains **d** and **e** . Indeed, for both compatibility regimes the allocation in these domains is such, that $m^y = 1$. In this case BCx regime has, clearly, no positive effect on the incentives of new x-agents, and hence NC regime is optimal.

▷ *Domain* **a**

Here the equilibrium is of the type E_{0000} in both NC and BCx regimes. Let us denote

$$K_1 = 1 - b^x b^y (s-1)^2 - s^2(b^y c^x + c^x c^y + b^x c^y - b^x b^y c^x c^y),$$
$$K_2 = 1 - b^x b^y (s-1)^2 - b^y c^x (s-1)^2 - s^2(c^x c^y + b^x c^y).$$

Notice, that due to Assumption 4, $K_1 > 0, K_2 > 0$. The equilibrium prices and demands in NC regime are as follows:

$$m^x = \frac{B}{2} \cdot \frac{1 + c^y s + b^y(s - 1 - c^x s)}{K_1}, \qquad m^y = \frac{B}{2} \cdot \frac{1 + c^x s + b^x(s - 1 - c^y s)}{K_1},$$
$$n^x = \frac{B}{2} \cdot \frac{1 + b^y s + c^y s + b^x b^y(s - 1 - c^y s)}{K_1}, \quad n^y = \frac{B}{2} \cdot \frac{1 + b^x s + c^x s + b^x b^y(s - 1 - c^x s)}{K_1}$$
$$A_0^x = A_0^y = A_1^x = A_1^y = \frac{B}{2}.$$

In BCx regime the prices and allocations are

$$m^x = \frac{B[1 + b^y(s-1) + c^y s] + b^y c^x [b^y(s-1)^2 + c^y s^2]}{2K_2},$$
$$m^y = \frac{B[1 + b^x(s-1) + c^x(s-1) - c^y s(b^x + c^x)] + b^y c^x (s-1)}{2K_2},$$
$$n^x = \frac{B[1 + b^y(s-1) + c^y s] + b^y[1 - b^x b^y (s-1)^2 - b^x c^y s^2]}{2K_2},$$
$$n^y = \frac{B[1 + b^y c^x (s-1) + c^x s + b^x b^y (s-1) + b^x s] + b^y c^x s}{2K_2},$$
$$A_0^x = A_0^y = A_1^y = \frac{B}{2}, A_1^x = \frac{B + b^y}{2}.$$

Calculating profits Π^{NC} and Π^{BCx} and comparing them for $b^x \to 0, b^y \to 0$, it is easy to establish, that $NC \prec BCx$ if

$$c^y < \frac{1 - B[1 + c^x(s-1)]}{c^x(s-1)^2} \equiv g_2(c^x).$$

Note, that $g_2(c^x) < h_1$ if $c^x > (1-B)/(2s-1)$

$\triangleright$ *Domain* **b**
Comparison of equilibrium profits in domain **b** leads to the conclusion, that $\Pi^{NC} < \Pi^{BCx}$ if $B - 1 + 2b^x(s-1) + 2c^x s < 0$. Given $b^x \to 0$ the inequality is satisfied if $c^x < \frac{1-B}{2s-1}$.

$\triangleright$ *Domain* **c**
In domain **c**, $\Pi^{NC} > \Pi^{BCx}$ is equivalent to

$$4(c^x)^2(s-1)^2 + c^x[4 + 4B(s-1) - 8s] + 4 - 4B + B^2 < 0.$$

Let us denote the left-hand side of the inequality as $\varphi(c^x)$. Since $\varphi(c^x)$ is a quadratic, convex function, $\varphi(c^x) < 0$ if $c_1 < c^x < c_2$, where

$$c_{1,2} = \frac{-1 - B(s-1) + 2s \mp \sqrt{4s - 3 + 2B - 2Bs}}{2(s-1)^2}$$

In addition, domain **c** is feasible if $\frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$. It is easy to see, that $c_2 > \frac{2-B}{2(s-1)}$ and $c_1 < \frac{2-B}{2s}$. Hence, $BCx \prec NC$ for any $\frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$.

$\triangleright$ *Domain* **f**
This domain is feasible, if $h_2 < c^y < h_3$. Let us denote $\psi(b^y) = \Pi^{NC} - \Pi^{BCx}$. From the specification of demand functions (see Section 4), it is clear, that if $b^y = 0$, then BCx regime is equivalent to NC regime in terms of equilibrium demands, prices and monopolist's profits. Therefore, in order to establish whether $\Pi^{NC} > \Pi^{BCx}$ for b^y and b^x in the neighborhood of zero, we will investigate the derivative of $\psi(b^y)$. In particular,

$$\frac{\partial \psi}{\partial b^y}|_{c^y = h_2} = \frac{(2-B)(2s-B)}{4s^2} > 0, \qquad \frac{\partial \psi}{\partial b^y}|_{c^y = h_3} = \frac{(2-B)}{2s-1} > 0.$$

Therefore, $BCx \prec NC$ for any $c^y = h_2$ and $c^y = h_3$. It follows that $BCx \prec NC$ also for $h_2 < c^y < h_3$.

Summarizing the results, we conclude, that $NC \prec BCy$ for any $c^x < \frac{1-B}{2s-1}$ or $c^y < g_2(c^x)$. □

Proof of Proposition 6. Since $b^y = 0$, we will omit the second subscript in the notations for equilibria we have used so far. The candidates for equilibrium allocations, feasibility and optimality conditions are summarized in Table 8 in Appendix B. The derivation of equilibrium allocations is analogical to the case of mature and emerging market and is therefore omitted. Again, let us start with some notation:

$$h_1 = \frac{2-B}{s[B+2s(b^x+c^x)]}, \qquad h_2 = \frac{2-B-Bb^xs-Bc^xs}{2s^2(b^x+c^x)},$$
$$h_3 = \frac{2-B}{s(B+2c^xs)+b^x[2+B(s-1)-3s-2s^2]},$$
$$h_4 = \frac{2-b^x-B[1+b^x(s-1)+c^xs]}{2[b^x(s-1)^2+c^xs^2]}, \qquad h_5 = \frac{2-B}{(s-1)[B+2c^xs+b^x(2s-1)]},$$
$$h_6 = \frac{2-b^x-B[1+b^x(s-1)]-2c^xs}{2b^x(s-1)^2}.$$

In addition, let $k_1 = \frac{2-B-2b^xs}{2s}$ and $k_2 = \frac{2-B-2b^xs+b^x}{2s}$. If we fix B, s and b^x then, using optimality conditions in Table 8, the whole range of parameters (c^x, c^y) can be divided into several domains. These domains are summarized in the Table 3 and are illustrated on Figure 7 in Appendix B. We split the proof of the proposition in a number of cases.

▷ *Domains* **i** *and* **k**

From the table we can readily see that in domains **i** and **k**, $BCy \prec NC$. Indeed, in both domains equilibrium allocation is such, that $m^x = 1$. In this case BCy does not improve incentives of new agents of type y and hence has no advantages compared with NC regime.

▷ *Domain* **l**

Calculating the respective profits and comparing them we receive, that

$$\Pi^{NC} > \Pi^{BCy} \iff B^2 + 4B[-1+c^y(s-1)] + 4[1+c^y+(c^y)^2(s-1)^2-2c^ys] < 0.$$

Let us denote the expression on the left-hand side of the inequality as $\varphi_1(c^y)$, which is a quadratic, convex function of c^y. Hence, $\varphi_1(c^y) < 0$ if $c_1 < c^y < c_2$, where

$$c_{1,2} = \frac{-1+B+2s-Bs \mp \sqrt{-3+2B+4s-2Bs}}{2(s-1)^2}.$$

In addition, feasibility constrains for domain **l** require $\frac{2-B}{2s} < c^y < \frac{2-B}{2(s-1)}$. Since $c_1 < \frac{2-B}{2s}$ and $c_2 > \frac{2-B}{2(s-1)}$, for all range of parameters in domain **l**, $\varphi_1(c^y) < 0$, which implies $BCy \prec NC$.

▷ *Domains* **j**, **g** *and* **h**

domain	feasibility condition	type of allocation NC	BCy
a	$c^y < \min[h_1, h_2, h_4]$	E_{000}	E_{000}
b	$h_2 < c^y < h_4$	E_{000}	E_{001}
c	$\max[h_2, h_4] < c^y < \frac{2-B}{2s}$	E_{001}	E_{001}
d	$\max[\frac{2-B}{2s}, h_4] < c^y < h_2$	E_{001}	E_{000}
e	$\frac{2-B}{2s} < c^y < h_4, c^x > k_1$	E_{111}	E_{000}
f	$h_1 < c^y < h_3, c^x < k_1$	E_{110}	E_{000}
g	$h_3 < c^y < h_5, c^x < k_1$	E_{110}	E_{010}
h	$h_3 < c^y < [h_5, h_6], c^x > k_1$	E_{111}	E_{010}
i	$h_5 < c^y, c^x < k_1$	E_{110}	E_{110}
j	$h_5 < c^y, k_1 < c^x < k_2$	E_{111}	E_{000}
k	$cy > \frac{2-B}{2s}, c^x > k_2$	E_{111}	E_{111}
l	$\max[\frac{2-B}{2s}, h_6] < c^y < \frac{2-B}{2(s-1)}$	E_{111}	E_{011}

Table 3: Equilibrium allocations on the asymmetric market

The inequality $\Pi^{NC} > \Pi^{BCy}$ is equivalent to

$$4 - 4B + B^2 + (b^x)^2(1-2s)^2 - 8b^x s+$$
$$+4(c^x)^2 s^2 + Bb^x(4s-2) + c^x[4s(B-2) + 4b^x s(2s-1)] < 0.$$

Let us denote an expression on the left-hand side of the inequality as $\varphi_2(c^x)$, which is a quadratic, convex function of c^x. Hence, $\varphi_2(c^x) < 0$ if $c_1 < c^x < c_2$, where

$$c_{1,2} = \frac{2 - B \mp 2\sqrt{b^x} + b^x - 2b^x s}{2s}.$$

In addition, feasibility constrains requires that $k_1 < c^x < k_2$. It is easy to check, that $c_1 < k_1$ and $c_2 > k_2$. Hence, $\varphi_2(c^x) < 0$ in domain **j**, which implies $BCy \prec NC$.

Using very similar line of argument it is easy to show, that $BCy \prec NC$ also in domains **g** and **h**.

$\triangleright$ *Domain* **b**

Consider now domain **b**, which is feasible if $h_2 < c^y < h_4$. Observe, that $h_2 < h_4$ if $c^x > \frac{4s-2-b^x s^2 + B[1-(2-b^x)s - b^x s^2]}{s[B(s-1)+s]} \equiv k_3$. Notice further, that h_2 and h_4 are decreasing functions, and the value of these functions at $c^x = k_3$ is $(1-B)/[2(2s-1)]$. Hence, everywhere in domain **b**, $c^y < (1-B)/[2(2s-1)]$. Finally, $h_2 < c^y < h_4$ is equivalent to

$$\frac{2 - b^x(1 + 2c^y(s-1)^2) + B(-1 + b^x + b^x s)}{s(B + 2c^y s)} < c^x < \frac{2 - B - Bb^x s - 2b^x c^y s^2}{s(B + 2c^y s)}.$$

Calculating the profits and comparing them, we receive, that

$$\Pi^{NC} < \Pi^{BCy} \iff \frac{\varphi_3(c^x)}{4[-1 + b^x(c^y)^2 + c^x c^y s^2]} < 0$$

(the expression for function φ_3 is not listed here as it is rather complicated). Due to Assumption 4, we have $b^x(c^y)^2 + c^x c^y s^2 < 1$. Hence, $\Pi^{NC} < \Pi^{BCy}$ if and only if $\varphi_3(c^x) > 0$. Function $\varphi_3(c^x)$ is quadratic in c^x with coefficient on $(c^x)^2$ being $-B^2 c^y s^2 - 4B(c^y)^2 s^3 - 4(c^y)^3 s^4 < 0$. Hence, $\varphi_3(c^x) > 0$ if and only if $c_1 < c^x < c_2$:

$$c_{1,2} = -\frac{1}{s(B + 2c^y s)^2}\big[B^2(1 + b^x s) + 2B\big(-1 + c^y s(1 - b^x + 2b^x s)\big) + \\ + 2c^y s\big(-2 + b^x(1 + c^y - 2c^y s + 2c^y s^2)\big) \pm \sqrt{K_1}\big].$$

where $K_1 = -b^x[-1 + B + c^y(2s-1)]\,[b^x(c^y)^2 s^2(1 + c^y - 2c^y s) + B^2(1 + c^y s) + Bc^y s(2 + (2 - b^x)c^y s)]$.

It is left to verify that $c_1 < \frac{2 - b^x(1 + 2c^y(s-1)^2) + B(-1 + b^x + b^x s)}{s(B + 2c^y s)}$ and $c_2 > \frac{2 - B - Bb^x s - 2b^x c^y s^2}{s(B + 2c^y s)}$. Both this inequalities hold if $c^y < \frac{1-B}{2s-1}$, which is true for any $c^y < \frac{1-B}{2(2s-1)}$. Hence, in domain **b**, $NC \prec BCy$.

▷ *Domain* **a**
In domain **a** the profits in NC and BCx regime are as follows:

$$\Pi^{NC} = \frac{B^2\big[c^x + c^y + 2c^x c^y s + b^x(1 + 2c^y s)\big]}{4(1 - b^x c^y s^2 - c^x c^y s^2)},$$

$$\Pi^{BCy} = \frac{1}{4(1 - b^x c^y s^2 - c^x c^y s^2)}\big[(b^x)^2 c^y + 2Bb^x c^y\big(1 + b^x(s-1) + c^x s\big) + \\ + B^2\big(c^x + c^y + 2c^x c^y s + b^x(1 - 2c^y - c^x c^y + 2c^y s)\big)\big].$$

Comparing profits we receive, that $NC \prec BCy$ if

$$c^y < \frac{(B-1)[t - c^x + B(2 + c^x - t + 2st)]}{s^2(c^x - t)t - 2Bs^2 t[1 + c^x + t(s-1)] + B^2[1 + 2s(t-1) - (2 - c^x)s^2 t]},$$

where $t \equiv b^x + c^x$. Let us denote the right-hand side of this inequality as $\mathcal{I}_1$.

▷ *Domain* **d**
In domain **d** feasibility constrains require $h_2 < c^x < h_4$, which is equivalent to

$$\frac{2 - B - Bb^x s - 2b^x c^y s^2}{s(B + 2c^y s)} < c^x < \frac{2 - b^x[1 + 2c^y(s-1)^2] + B(-1 + b^x + b^x s)}{s(B + 2c^y s)}.$$

Comparing profits of the monopolist, we establish that $\Pi^{NC} < \Pi^{BCy}$ if

$$\frac{\varphi_4(c^x)}{-1 + b^x c^y(s-1)^2 + c^x c^y s^2} < 0.$$

The denominator in the above expression in negative by Assumption 4. The nominator $\varphi_4(c^x)$ is a quadratic function where the coefficient on $(c^x)^2$ equals $s^2(B+2c^ys)^2$. Hence, $\varphi_4(c^x) > 0$ if $c^x < c_1$ or $c^x > c_2$:

$$\begin{aligned} c_{1,2} = -\ &\big[B^2\big(1+b^x(s-1)\big) + 2c^ys\big(-2+b^xc^y(1-2s+2s^2)\big)+ \\ &B\big(-2+2c^ys+b^x(1+c^y(2-4s+4s^2))\big) \mp 2\sqrt{K_2}\big]/[s(B+2c^ys)^2]. \end{aligned}$$

where

$$\begin{aligned} K_2 = b^x[-1+B+c^y(2s-1)]\,&\big[B^2\big(1+b^xc^y(s-1)+c^ys\big) \\ &+ Bc^ys\big(2+b^x+2c^ys-b^xc^y(3s-2)\big) + b^x(c^y)^2s^2\big(1+c^y(2s-1)\big)\big]. \end{aligned}$$

$K_2 > 0$ if $c^y > \frac{1-B}{2s-1}$. Further, $c_2 > \frac{2-b^x(1+2c^y(s-1)^2)+B(-1+b^x+b^xs)}{s(B+2c^ys)}$ for $c^y > \frac{1-B}{2s-1}$. Hence, $\varphi_4(c^y) > 0$ if $c^x < c_1$. Let us denote $\varphi_4^{-1}(c^y)$ as $\mathcal{I}_2$.

▷ *Domains* **e** *and* **f**

Using the same argument, we can show that $NC \prec BCy$ if $c^y < \mathcal{I}_4$ in domain **e** and $NC \prec BCy$ if $c^y < \mathcal{I}_5$ in domain **f**. Expressions for $\mathcal{I}_4$ and $\mathcal{I}_5$ can be easily derived as the roots of the quadratic equation (similar as in previous case). The expressions, however, are rather complicated and therefore are not presented here.

▷ *Domain* **c**

Finally, consider domain **c**. Calculating the respective profits and comparing them we receive, that $\Pi^{NC} < \Pi^{BCy}$ if $B-1+c^y(2s-1) < 0$, which is equivalent to $c^y < \frac{1-B}{2s-1}$. Let us denote $\mathcal{I}_3 \equiv \frac{1-B}{2s-1}$.

Combining the results we can describe a curve $\mathcal{I}^y$ as a function of c^x which shows the indifference of the monopolist between making technologies compatible versus not compatible with each other.

$$\mathcal{I}^y = \begin{cases} \mathcal{I}_1, & \text{if } \mathcal{I}_1 < \min[h_1, h_2, h_4], \\ \mathcal{I}_2, & \text{if } h_2 < \mathcal{I}_2 < \min[\frac{2-B}{2s}, h_4], \\ \mathcal{I}_3, & \text{if } \frac{1-B}{2s-1} > h_4, \\ \mathcal{I}_4, & \text{if } \frac{2-B}{2s} < \mathcal{I}_4 < h_3 \text{ and } k_1 < c^x < k_2, \\ \mathcal{I}_5, & \text{if } h_3 < \mathcal{I}_5 < h_1 \text{ and } c^x < k_1. \end{cases}$$

We have shown, that $BCy \prec NC$ in domains **g**, **h**, **i**, **j**, **k**, **l** and $NC \prec BCy$ in domain b. Further, for each domain **a**, **c**, **d**, **e**, **f** there is a single indifference curve, such that $NC \prec BCy$ below this curve and the inequality is reversed otherwise. It remains to show, that for each c^x there is a unique cutoff value of c^y, such that below this value $NC \prec BCy$ and above this value the inequality is reversed.

Assume, to the contrary, that for some $c^x = \hat{c}$ there are two distinct values of c^y, $c^y = c_1$ and $c^y = c_2$, such that $NC \prec BCy$ for all $c^y < c_1$ and $c^y < c_2$ and $BCy \prec NC$ for all $c^y > c_1$ and $c^y > c_2$. Note that since for each domain there exist a unique indifference curve, points $(\hat{c}, c_1)$ and $(\hat{c}, c_2)$ must belong to the different domains. Assume first, that these domains have a common border (which is the case for domains **a** and **f**, **a** and **d**, **d** and **e**). Observe, that assumption $c_1 \neq c_2$ implies, that the indifference curve cannot be identical with the border between the domains. For clarity of notations, consider for example domains **a** and **f** (the argument is exactly the same for other pairs of domains).

The border between two domains gives the values of (c^x, c^y), such that the feasibility constrains for both domains are satisfied in the limit cases. Hence, if we denote the difference of profits in domain **a** as $\psi^a \equiv \Pi^{NC} - \Pi^{BCx}$ and in domain **f** as $\psi^f \equiv \Pi^{NC} - \Pi^{BCx}$, then on the border between two domains must hold $\psi^a \equiv \psi^f$. Then the existence of distinct c_1 and c_2 is possible only if $\psi^a = \psi^f = 0$. But this contradicts the assumption, that there is a unique indifference curve for each domain.

Consider now the case, where c_1 and c_2 belong to the two domains which do not have a common border (domain **a** and **e**). Then the function $\psi \equiv \Pi^{NC} - \Pi^{BCx}$ must change sign on the border of the domain **d**. Since domains **e** and **d** are neighbor domains, see discussion above. □

Proof of Corollary 3. If the parameters of the model are such, that interior solution is feasible, then the indifference curve is given by $\mathcal{I}^y = \mathcal{I}_1$, where $\mathcal{I}_1$ is given in the proof of Proposition 6.

First we prove that the indifference curve is downward sloping. Consider the derivative $\partial\mathcal{I}_1/\partial c^x = \varphi_1(B)/[\psi_1(B)]^2$, where

$$\psi_1(B) = b^x s^2 t + 2Bs^2 t(1 - b^x + st) + B^2[-1 - 2s(t-1) + s^2(2 + b^x - t)t].$$

Hence, $\partial\mathcal{I}_1/\partial c^x < 0$ is equivalent to $\varphi_1(B) < 0$. Function $\varphi_1(B)$ is polynomial of the form $\varphi_1(B) = \alpha_0 B^4 + \alpha_1 B^3 + \alpha_2 B^2 + \alpha_3 B + \alpha_4$, where $\alpha_i, i \in \{1,2,3,4\}$ are coefficients that depend on b^x, c^x, and s. The function $\varphi_1(B)$ has the following properties:

$$\varphi_1(0) = -(b^x s) < 0, \qquad \varphi_1'(0) = -4b^x s^2[1 + b^x(s-1) + c^x s] < 0,$$
$$\varphi_1(1) = 0, \qquad \varphi_1'(1) = 2s(2s-1)(1 + b^x s + c^x s)^2 > 0.$$

Hence, to prove that $\varphi_1(B) < 0$ for any $B \in (0,1)$ it is sufficient to show, that $\varphi_1''(B)$ changes sign at most once on the interval $[0,1]$. This is indeed the case, since $\varphi_1''(B)$ is a quadratic function which has the following properties:

$$\varphi_1''(0) = 4s[-2(b^x)^2(s-1)^2 s - 2s(1 + c^x s)^2 - b^x(1 + c^x s)(4s^2 - 4s - 1)] < 0,$$
$$\varphi_1''(1) = 4s[(4s-3)(1 + c^x s)^2 + (b^x)^2 s(1 - 5s + 4s^2) + b^x(1 + c^x s)(1 - 8s + 8s^2)] > 0.$$

Thus, $\varphi_1(B) < 0$ on the interval $B \in (0,1)$, which implies that $\partial\mathcal{I}_1/\partial c^x < 0$.

Second, we prove the statement for $\bar{s}$. Towards this end consider the derivative $\partial\mathcal{I}_1/\partial s = \varphi_2(B)/[\psi_1(B)]^2$. By the same argument as above, $\partial\mathcal{I}_1/\partial s < 0$ is equivalent to $\varphi_2(B) < 0$. Function $\varphi_2(B)$ is polynomial of the form $\varphi_2(B) = \beta_0 B^4 + \beta_1 B^3 + \beta_2 B^2 + \beta_3 B + \beta_4$, where $\beta_i, i \in \{1,2,3,4\}$ are coefficients which depend on b^x, c^x and s. The function $\varphi_2(B)$ has the following properties:

$$\varphi_2(0) = -2(b^x)^2 st < 0, \qquad \varphi_2'(0) = -8b^x st(1 - b^x + st) < 0,$$
$$\varphi_2(1) = 0, \qquad \varphi_2'(1) = 2(1 + st)^2[2 + (4s-1)t] > 0.$$

Given this properties, to prove that $\varphi_2(B) < 0$ for any $B \in (0,1)$ it is sufficient to show, that $\varphi_2''(B)$ changes sign at most once on the interval $[0,1]$. This is indeed the case, since $\varphi_2''(B)$ is a quadratic function which has the following properties. The function is concave, since the coefficient at B^2 is

$$b^x st[b^x - t(1+s)] - b^x t - (2-t)(1+st)^2 < 0,$$

which is negative for any $t = b^x + c^x < 2$. Further $\varphi_2''(1) < 0$ for any $b^x < 1$:

$$\varphi_2''(1) = -4(1+st)\big[-6 + (3-14s)t + (3-8s)st^2 + b^x\big(1+(3s-1)t\big)\big] < 0.$$

Hence, $\varphi_2(B) < 0$ on the interval $B \in (0,1)$, which implies that $\partial \mathcal{I}_1/\partial s < 0$.

Third, we prove the statement for $\bar{B}$. For any $s > 1$ and $B \in (0,1)$:

$$\frac{\partial \mathcal{I}^y}{\partial B} = -\frac{2B(2s-1)(2+st)^2[B + b^x(1-B) + Bst]}{[\psi_1(B)]^2} < 0.$$

Fourth, the proof of the statement for $\bar{b}^x$ follows the same logic as the proof of the first and the second statement and is therefore abandoned. □

B Appendix: Tables and figures

$\tilde{\beta}_1$	$\mathrm{d}m^x/d\gamma^x$	$c^x[(s-1)(s-\gamma^x)b^y H''(n^y) + s(s-\gamma^y)c^y H''(m^y)]$
	$\mathrm{d}m^y/\mathrm{d}\gamma^x$	$c^x[(s-\gamma^x)H''(m^x)H''(n^y) + (s-\gamma^y)\Gamma b^x c^y]$
	$\mathrm{d}n^x/\mathrm{d}\gamma^x$	$H''(m^x)H''(m^y)H''(n^y) - (s-1)^2 b^x b^y H''(n^y) - (s-\gamma^y)^2 b^x c^y H''(m^y)$
	$\mathrm{d}n^y/\mathrm{d}\gamma^x$	$c^x[sH''(m^x)H''(m^y) - (s-1)\Gamma b^x b^y]$
$\tilde{\beta}_2$	$\mathrm{d}m^x/d\gamma^x$	$b^y[-(s-1)H''(n^x)H''(n^y) + s\Gamma c^y c^y]$
	$\mathrm{d}m^y/\mathrm{d}\gamma^x$	$-H''(m^x)H''(n^x)H''(n^y) + s^2 c^x c^y H''(m^x) + (s-\gamma^y)^2 b^x c^y H''(n^x)$
	$\mathrm{d}n^x/\mathrm{d}\gamma^x$	$b^y[-(s-\gamma^y)^2 H''(m^x)H''(n^y) - (s-\gamma^y)\Gamma b^x c^y]$
	$\mathrm{d}n^y/\mathrm{d}\gamma^x$	$b^y[-s(s-\gamma^x)c^x H''(m^x) - (s-1)(s-\gamma^y)b^x H''(n^x)]$

Table 4: Coefficients $\tilde{\beta}_1$ and $\tilde{\beta}_2$

E_{0000}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{0010}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x = 1$	$n^y \in (0,1)$
E_{0011}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x = 1$	$n^y = 1$
E_{0001}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y = 1$
E_{1000}	$m^x = 1$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{1010}	$m^x = 1$	$m^y \in (0,1)$	$n^x = 1$	$n^y \in (0,1)$
E_{1011}	$m^x = 1$	$m^y \in (0,1)$	$n^x = 1$	$n^y = 1$
E_{1001}	$m^x = 1$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y = 1$
E_{0100}	$m^x \in (0,1)$	$m^y = 1$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{0110}	$m^x \in (0,1)$	$m^y = 1$	$n^x = 1$	$n^y \in (0,1)$
E_{0111}	$m^x \in (0,1)$	$m^y = 1$	$n^x = 1$	$n^y = 1$
E_{0101}	$m^x \in (0,1)$	$m^y = 1$	$n^x \in (0,1)$	$n^y = 1$
E_{1100}	$m^x = 1$	$m^y = 1$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{1110}	$m^x = 1$	$m^y = 1$	$n^x = 1$	$n^y \in (0,1)$
E_{1111}	$m^x = 1$	$m^y = 1$	$n^x = 1$	$n^y = 1$
E_{1101}	$m^x = 1$	$m^y = 1$	$n^x \in (0,1)$	$n^y = 1$

Table 5: Candidates for equilibrium allocations with linear demand function

	NC		*BCx*	
	feasibility conditions	optimality conditions	feasibility conditions	optimality conditions
E_{0000}	$b^y < \min[h_1, h_2]$	$b^y < \min[h_1, h_2]$	$b^y < \min[h_2, h_5]$	$b^y < \min[h_2, h_5]$
E_{0010}	$b^y < \min[h_2, h_3]$	$h_1 < b^y < \min[h_2, h_3]$	$b^y < \min[h_2, h_3]$	$h_5 < b^y < \min[h_2, h_3]$
E_{0011}	$b^y < \min[h_3, h_4]$	$\max[h_1, h_2] < b^y < \min[h_3, h_4]$	$b^y < \min[h_3, h_4]$	$\max[h_2, h_5] < b^y < \min[h_3, h_4]$
E_{0001}	$b^y < \min[h_1, h_4]$	$h_2 < b^y < \min[h_1, h_4]$	$b^y < \min[h_4, h_5]$	$h_2 < b^y < \min[h_4, h_5]$
E_{1000}	$b^y < \frac{2-B}{s[B+2b^x(s-1)]}, b^x < \frac{2-B}{2s}$	$\times$	$b^y < \frac{2-B}{1+B(s-1)+2b^x(s-1)^2}, b^x < \frac{2-B}{2s}$	$\times$
E_{1010}	$b^x < \frac{2-B}{2s}$	$b^y > h_3, b^x < \frac{2-B}{2s}$	$b^x < \frac{2-B}{2s}$	$b^y > h_3, b^x < \frac{2-B}{2s}$
E_{1011}	$b^x < \frac{2-B}{2(s-1)}$	$b^y > h_3, \frac{2-B}{2s} < b^x < \frac{2-B}{2(s-1)}$	$b^x < \frac{2-B}{2(s-1)}$	$b^y > h_3, \frac{2-B}{2s} < b^x < \frac{2-B}{2(s-1)}$
E_{1001}	$b^y < \frac{2-B}{s[B+2b^x(s-1)]}, b^x < \frac{2-B}{2(s-1)}$	$\times$	$b^x < \min[\frac{2-B}{2(s-1)}, \frac{2-B}{1+B(s-1)+2b^x(s-1)^2}]$	$\times$
E_{0100}	$b^y < \min[\frac{2-B}{2s}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$	$b^y < \min[\frac{2-B}{2s-1}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$
E_{0110}	$b^y < \min[\frac{2-B}{2(s-1)}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$	$b^y < \min[\frac{2-B}{2(s-1)}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$
E_{0111}	$b^y < \frac{2-B}{2(s-1)}$	$\max[\frac{2-B}{2s}, h_4] < b^y < \frac{2-B}{2(s-1)}$	$b^y < \frac{2-B}{2(s-1)}$	$\max[\frac{2-B}{2s-1}, h_4] < b^y < \frac{2-B}{2(s-1)}$
E_{0101}	$b^y < \frac{2-B}{2s}$	$h_4 < b^y < \frac{2-B}{2s}$	$b^y < \frac{2-B}{2s-1}$	$h_4 < b^y < \frac{2-B}{2s-1}$
E_{1100}	$\frac{2-B}{2s} < \min[b^x, b^y]$	$\times$	$b^y < \frac{2-B}{2s-1}, b^x < \frac{2-B}{2s}$	$\times$
E_{1110}	$b^x < \frac{2-B}{2s}$	$\times$	$b^x < \frac{2-B}{2s}$	$\times$
E_{1111}	$b^x \in [0, \infty), b^y \in [0, \infty)$	$\frac{2-B}{2(s-1)} < \min[b^x, b^y]$	$b^x \in [0, \infty), b^y \in [0, \infty)$	$\frac{2-B}{2(s-1)} < \min[b^x, b^y]$
E_{1101}	$b^y < \frac{2-B}{2s}$	$\times$	$b^y < \frac{2-B}{2s-1}$	$\times$

Table 6: Mature market: feasibility and optimality conditions for equilibrium candidates in NC and BCx regimes

	NC		BCx	
	feasibility conditions	optimality conditions	feasibility conditions	optimality conditions
E_{0000}	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$
E_{0010}	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$
E_{0011}	$\frac{2-B}{2s} > \max[c^x, c^y]$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$
E_{0001}	$c^y < \min[h_2, \frac{2-B}{2s}]$	$\times$	$c^y < \min[\frac{2-B}{2s}, h_3]$	$h_2 < c^y < \min[\frac{2-B}{2s}, h_3]$
E_{1000}	$c^y < \min[h_1, h_2]$	$\times$	$c^y < \min[h_1, h_2]$	$\times$
E_{1010}	$c^x < \frac{2-B}{2s}$	$c^y > h_1, c^x < \frac{2-B}{2s}$	$c^x < \frac{2-B}{2s}$	$c^x < \frac{2-B}{2s}, c^y > h_1$
E_{1011}	$c^x < \frac{2-B}{2s}$	$\times$	$c^x < \frac{2-B}{2(s-1)}$	$c^y > \frac{2-B}{2s}, \frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$
E_{1001}	$c^y < \min[h_2, \frac{2-B}{2s}]$	$\times$	$c^y < \min[\frac{2-B}{2s}, h_3]$	$\times$
E_{0100}	$c^y < \min[h_1, h_2]$	$\times$	$c^y < [h_1, h_2]$	$\times$
E_{0110}	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$
E_{0111}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$
E_{0101}	$c^y < \frac{2-B}{2s}$	$h_2 < c^y < \frac{2-B}{2s}$	$c^y < \frac{2-B}{2s}$	$h_3 < c^y < \frac{2-B}{2s}$
E_{1100}	$c^y < \min[h_1, h_2]$	$\times$	$c^y < \min[h_1, h_2]$	$\times$
E_{1110}	$c^x < \frac{2-B}{2s}$	$\times$	$c^x < \frac{2-B}{2s}$	$\times$
E_{1111}	$b^x \in [0, \infty), b^y \in [0, \infty)$	$\frac{2-B}{2s} < \min[c^x, c^y]$	$b^x \in [0, \infty), b^y \in [0, \infty)$	$c^y > \frac{2-B}{2s}, c^x > \frac{2-B}{2(s-1)}$
E_{1101}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$

Table 7: New market: feasibility and optimality conditions for equilibrium candidates in NC and BCx regimes

	NC		*BCy*	
	feasibility conditions	optimality conditions	feasibility conditions	optimality conditions
E_{000}	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$	$c^y < \min[h_3, h_4]$	$c^y < \min[h_3, h_4]$
E_{010}	$c^y < \min[h_1, \frac{2-B-Bb^x s-2c^x s}{2b^x s^2}]$	$\times$	$c^y < \min[h_5, h_6]$	$h_3 < c^y < \min[h_5, h_6]$
E_{011}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2(s-1)}$	$\max[\frac{2-B}{2s}, h_5] < c^y < \frac{2-B}{2(s-1)}$
E_{001}	$c^y < \frac{2-B}{2s}$	$h_2 < c^y < \frac{2-B}{2s}$	$c^y < \frac{2-B}{2s}$	$h_4 < c^y < \frac{2-B}{2s}$
E_{100}	$c^y < \min[h_1, \frac{2-B(1+c^x s)-2b^x s}{2c^x s^2}]$	$\times$	$c^y < \min[\frac{2-B}{s(B+2c^x s+b^x(2s-1))}, \frac{2-B(1+c^x s)-b^x(2s-1)}{2c^x s^2}]$	$\times$
E_{110}	$c^x < \frac{2-B-2b^x s}{2s}$	$c^y > h_1, c^x < \frac{2-B-2b^x s}{2s}$	$c^x < \frac{2-B-b^x(2s-1)}{2s}$	$c^y > h_6, c^x < \frac{2-B-b^x(2s-1)}{2s}$
E_{111}	$c^x \in [0,\infty), c^y \in [0,\infty)$	$c^y > \frac{2-B}{2s}, c^x > \frac{2-B-2b^x s}{2s}$	$c^x \in [0,\infty), c^y \in [0,\infty)$	$c^y > \frac{2-B}{2(s-1)}, c^x > \frac{2-B-b^x(2s-1)}{2s}$
E_{101}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$

Table 8: Assymmetric market: feasibility conditions for equilibrium candidates

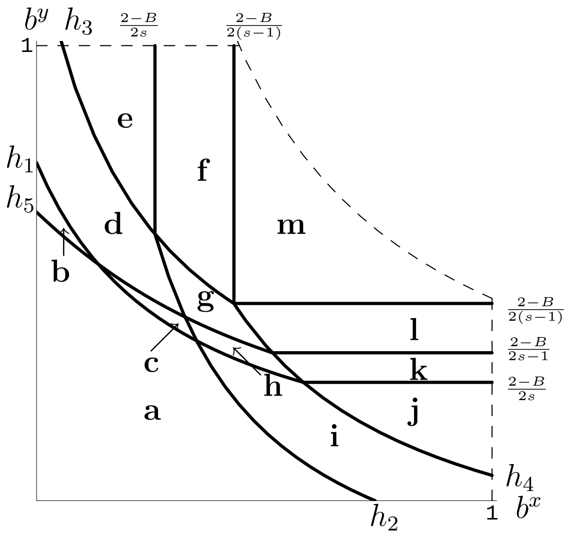


Figure 5: Equilibrium allocations for NC and BCx regimes at the mature market ($s = 2.5$, $B = 0.7$)

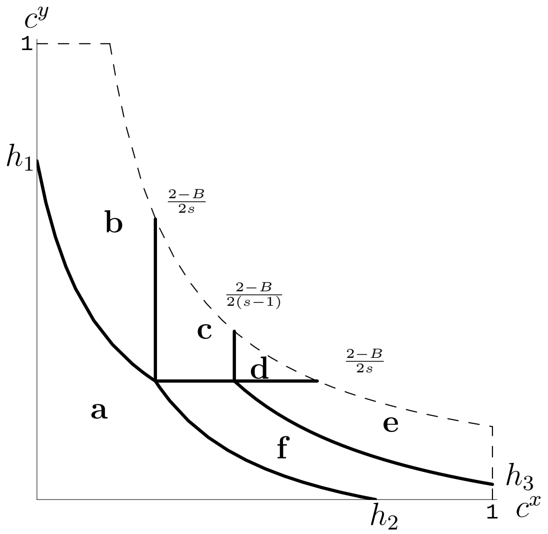


Figure 6: Equilibrium allocations for NC and BCx regimes at the emerging market ($s = 2.5$, $B = 0.7$)

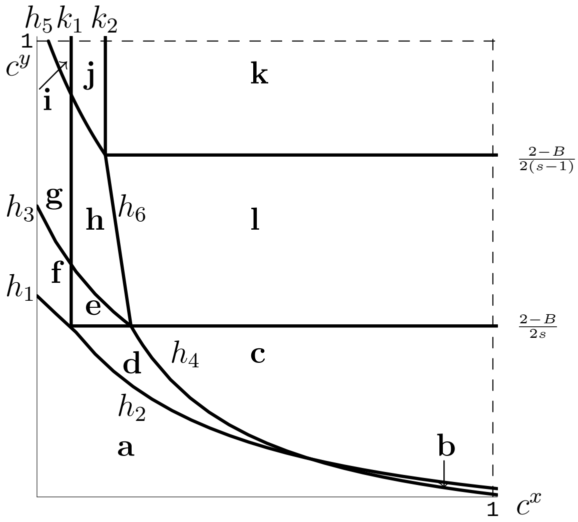


Figure 7: Equilibrium allocations for NC and BCy regimes at the asymmetric market ($s = 2$, $B = 0.5$, $b^x = .3$)

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