Hadron Collider Physics

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Exercises 1 – Formulae

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(a) Luminosity :

$$\mathcal{L} = f\frac{N_1 N_2 n_b}{4\pi\sigma^2}$$

where $\mathcal{L}$ $\hat{=}$Luminosity, f $\hat{=}$revolution frequency of a bunch, N_i $\hat{=}$number of particles per bunch, n_b $\hat{=}$number of colliding bunches and σ $\hat{=}$beam width.

(b) Energy loss of a circulating particle due to synchrotron radiation :

$$\Delta E = \hbar c \cdot \frac{4\pi\alpha}{3}\left(\frac{E}{m}\right)^4 \frac{1}{R}$$

where ΔE $\hat{=}$energy loss per turn and particle, α $\hat{=}$fine structure constant, E $\hat{=}$energy of the particle, m $\hat{=}$ mass of the particle, R $\hat{=}$radius of the circular path.

(c) The energy of particles (with the elementary charge q_e) which are circulating in a magnetic field B is given by:

$$E = 0.3\frac{\mathrm{GeV}}{\mathrm{T\,m}} BR$$

where R denotes the radius of circular flight path. The magnetic field is perpendicular to the flight path.

(d) Lorentz transformation (1D) :

$$\begin{pmatrix} \tilde{E} \\ \tilde{p} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix}$$

where E, p are the energy and the momentum, $\tilde{E}$, $\tilde{p}$ the transformed energy and momentum, $\beta = \frac{v}{c}$ and $\gamma^2 = (1-\beta^2)^{-1}$.

(e) Rapidity :

$$y = \frac{1}{2}\ln\frac{E + p_{\parallel}}{E - p_{\parallel}} = \frac{1}{2}\ln\frac{1+\beta\cos\theta}{1-\beta\cos\theta}$$

where y is the rapidity, E the energy of the particle and $p_{\parallel}$is the momentum component in the direction parallel to the beam axis and $E^2 = p_{\parallel}^2 + p_{\perp}^2 + m^2$, β the velocity in units of the velocity of light, and θ the polar angle of the particle with respect to the beam axis. All parameters are measured in the LAB frame.

(f) Pseudo Rapidity :

$$\eta = -\ln \tan \frac{\theta}{2}$$

where η is the pseudo rapidity and θ the polar angle with respect to the beam axis. The pseudo rapidity is a good approximation of the rapidity in the relativistic limit $\eta = \lim_{\beta \to 1} y$.

(g) Rapidity identities :

$$\cosh y = \frac{E}{\underbrace{\sqrt{p_\perp^2 + m^2}}_{E_\mathrm{T}}}$$

$$\sinh y = \frac{p_\parallel}{E_\mathrm{T}}$$

$$\tanh y = \frac{p_\parallel}{E} = \beta_\parallel$$

(h) Hyperbolic identities :

$$\cosh(a+b) = \cosh a \, \cosh b + \sinh a \, \sinh b$$

$$\tanh(a+b) = \frac{\tanh a + \tanh b}{1 + \tanh a \, \tanh b}$$

(i) Trigonometric identities :

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

(j) Constants :

(a) $\hbar c = 200\,\mathrm{MeV} \cdot \mathrm{fm}$, $c = 3 \cdot 10^8\,\mathrm{m\,s^{-1}}$

(b) $m_\mathrm{e} = .511\,\mathrm{MeV}$, $m_\mathrm{p} = 938\,\mathrm{MeV}$,$m_\mu = 106\,\mathrm{MeV}$