Rheinische Friedrich-Wilhelms-Universität Bonn Sommer Semester 2010

Hadron Collider Physics

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Exercises 1

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Classwork

1.1 Accelerators

1.1.1 Energy Loss at LEP

(a) The electron positron storage ring LEP was operated during its second phase (LEP2) at centre-of-mass energies up to 209 GeV. How much energy is needed (in MeV) to compensate the energy loss the electrons and positrons suffer in each turn due to synchrotron radiation (the circumference of the LEP/LHC tunnel is 26.7 km)?

(b) How many cavities are needed (cavity length 1 m, gradient 7 MV/m)?

1.1.2 TeV Scale Storage Rings.

(a) How much energy would be needed per turn and electron or positron for a storage ring with a centre-of-mass energy of 1 TeV?

(b) What about muons ? Is the required energy per turn for muons still too high (higher than the energy loss at LEP2) or why is there no muon collider yet ?

1.1.3 LHC

(a) What would be the maximum centre-of-mass energy of a proton proton collider when the energy loss per turn should not be larger than the energy loss per turn at LEP2 ?

(b) What would be the necessary magnetic field strength to keep the protons inside the tunnel of LEP (circumference= 26.7 km) ?

(c) What would be the maximum centre-of-mass energy if the magnetic field strength is limited to 10 T and how many 1 m long cavities with an average gradient of 7 MV/m are needed to compensate the resulting energy loss ?

1.2 Lorentz Transformation

1.2.1 Centre-of-mass energy

(a) What would be the centre-of-mass energy of a proton- anti-proton collider with proton and anti proton beams of 7 TeV?

(b) What would be the centre-of-mass energy if an electron at 100 GeV is used instead of the anti-proton?

(c) Cosmic rays have been observed with energies up to 10^{20} eV. How much energy do cosmic rays need to collide with protons in the atmosphere of the earth at LHC centre-of-mass energies ?

1.2.2 The Minkowski Inner Product

Show that

$$\boldsymbol{p}_1 \cdot \boldsymbol{p}_2 := E_1 E_2 - \sum_{i=1}^{3} p_{1,i} p_{2,i}$$

is Lorentz invariant.

1.3 Rapidity and Pseudo Rapidity

What is the error in percent when using the pseudo rapidity instead of the rapidity for a muon and a proton with a transverse momentum p_{T} of 1 GeV (5 GeV) and a pseudo rapidity η of 1 and 5.

1.4 Luminosity of the LHC

The nominal operation parameters of the LHC are $\sqrt{s} = 14\,\mathrm{TeV}$, circumference of 26.7 km, 2808 bunches per beam, 10^{11}protons per bunch, transverse width of the beam of 16.7 μm, 25 ns bunch spacing. What would be the nominal luminosity ?

Homework

1.5 Lorentz Group

(a) The Lorentz transformations form a group $\mathcal{L}$ with $\mathcal{L}_3 = \mathcal{L}_1 \otimes \mathcal{L}_2$. Show that

$$\mathcal{L}(\beta_3) = \mathcal{L}(\beta_1) \otimes \mathcal{L}(\beta_2) \quad \text{with} \quad \beta_3 = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$$

for all β_1, β_2 which are parallel $\boldsymbol{\beta_1} \parallel \boldsymbol{\beta_2}$ ($\beta = v/c$).

(b) Is the Lorentz group abelian ?

1.6 Rapidity

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}p_{\parallel}} = \frac{1}{E}$$

where $E = \sqrt{m^2 + p_{\parallel}^2 + p_{\perp}^2}$.

(b) Then show that the difference between two rapidities is invariant under Lorentz transformations:

$$\mathcal{L}(y_1) - \mathcal{L}(y_2) = y_1 - y_2$$

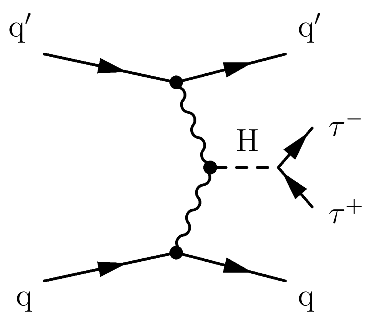
(c) Show that the invariant mass of a state which decays into particles 3 and 4 is given by:

$$m^2 \simeq 2E_T^2(\cosh(y_3 - y_4) - \cos(\phi_3 - \phi_4))$$

where y_i and ϕ_i denote the rapidity of the particles 3 and 4 and their angles in the plane perpendicular to the beam axis. The transverse energy of the two particles is approximately given by $E_{\mathrm{T}} \simeq E_i$. The mass of particles 3 and 4 is negligible with respect to E_{T}.

1.7 Higgs Boson Production at the LHC

(a) An important goal of the LHC is the discovery of the Higgs boson. Indirect methods suggest a light Higgs boson. A light Higgs boson of e.g. 120 GeV could be seen by searching for Higgs bosons decaying into τ leptons ($\mathrm{H} \to \tau^+\tau^-$). A promising channel is the so called vector boson fusion (VBF) process :



The cross section of this process is $\sigma_{\mathrm{qq}\to\mathrm{Hqq}} \times \mathrm{Br}(\mathrm{H} \to \tau^+\tau^-) = 4.3\,\mathrm{pb} \times 7\%$ at the nominal centre-of-mass energy ($\sqrt{\mathrm{s}} = 14\,\mathrm{GeV}$). How many Higgs bosons will be produced in one LHC "year" of 10^7s (assume the nominal LHC luminosity).

(b) In its initial phase the LHC was running with a single bunch per beam and experiment and with lower beam intensities (5×10^9 protons per bunch), and at $\sqrt{s} = 7\,\mathrm{TeV}$. The cross section at this centre-of-mass energy is $\sigma_{\mathrm{qq}\to\mathrm{Hqq}} = 1.2\,\mathrm{pb}$. How long would it take to produce the same amount of Higgs bosons ?